Physical **H**ydrology for **E**cosystems

BEE 3710

Some Useful Daily Environmental Energy Equations:

I. Solar Radiation

(I.1) $$Q_s = (1-A)(1-C_f)T_t S_o$$ [kJ m^{-2} day^{-1}]

T_t	=	atmospheric transmissivity (cloudy≈0.2, sunny≈0.75).	[-]
A	=	albedo (~0.05 for water, ~0.2 for vegetation, ~0.3 for soil, ~0.9 for snow)	[-]
C_f	=	Forest cover	[-]
S_o	=	the potential solar radiation	

(I.2) $$S_o = \frac{117500}{\pi}\left[\cos^{-1}\left(\tan(\delta)\tan(\lambda)\right)\sin(\lambda)\sin(\delta) + \cos(\lambda)\cos(\delta)\sin\left(\cos^{-1}\left(\tan(\delta)\tan(\lambda)\right)\right)\right]$$

[kJ m^{-2} day^{-1}]

λ	=	geographic latitude.	[radians]
δ	=	solar declination, or the latitude at which the sun is directly overhead	

(I.3) $$\delta = 0.4102\sin\left[\frac{\pi}{180}(J-80)\right]$$ [radians]

J	=	the day of the year (Julian date, e.g., Jan 1 = 1, Jan 2 = 2, … Dec 31 = 365)

Atmospheric Transmissivity (Campbell-Bristow eqn):

(I.4) $$T_t = 0.75\left[1-\exp\left(-0.036\exp(-0.154\overline{\Delta T})\Delta T^B\right)\right]$$ [-]

$\overline{\Delta T}$	=	Average temperature range 15 days before and after	[^{o}C]]
ΔT	=	Daily temperature range (T_{max} - T_{min})	[^{o}C]
B	=	Constant = 2.4	[-]

II. Long Wave Radiation

Stefan-Boltzmann Equation

(II.1) $$Q_l = \varepsilon\sigma(T_s + 273.15)^4$$ [kJ m^{-2} day^{-1}]

- T_s = temperature [^oC]
- ε = emissivity. [-]
- σ = Stephan-Boltzmann Constant, 4.90×10^{-6} [kJ m^{-2} $^oK^{-4}$ day^{-1}]
  (5.67×10^{-8} J m^{-2} $^oK^{-1}$ s^{-1})

<u>Terrestrial Long Wave Radiation (kJ m^{-2} day^{-1}):</u>

(II.2) $$Q_t = \varepsilon_s\sigma(T_s + 273.15)^4$$ [kJ m^{-2} day^{-1}]

- T_s = terrestrial surface temperature [^oC]
- ε_s = surface emissivity ≈ 0.95 - 0.98 [-]

<u>Atmospheric Long Wave Radiation (kJ m^{-2} day^{-1}):</u>

(II.3) $$Q_a = \varepsilon_{ac}\sigma(T_a + 273.15)^4$$ [kJ m^{-2} day^{-1}]

- T_a = the cloud or air temperature [^oC]
- ε_a = Clear sky emissivity $\varepsilon_a = 0.72 + 0.005T_a$ [-]
- ε_{ac} = Cloudy sky emissivity $\varepsilon_{ac} = (1 - 0.84C)\varepsilon_a + 0.84C$ [-]
  - C = cloudiness [fraction]

III. Sensible Heat Exchange

(III.1) $$Q_h \approx C_a(T_s - T_a)\left(\frac{f(u)}{\lambda}\right)$$ [kJ m^{-2} day^{-1}]

- T_s = temperature of the surface[^oC]
- T_a = temperature of the air [^oC]
- C_a = The heat capacity of air, 1.25 [kJ m^{-3} $^oC^{-1}$]
- λ = latent heat of vaporization, 2500 [kJ kg^{-1}]
- $f(u)$ = dependence of sensible heat exchange to windspeed, u [kJ m kg^{-1} day^{-1}]
  = $4.58 \times 10^5(1+u)$
- u = average daily windspeed [m s^{-1}]

The empirical term, $f(u)/\lambda$, is sometimes replaced by a more mechanistic expression for thermal conductance or resistance, $1/r_h$, where r_h has units [s/m]

(III.2) $$Q_h \approx C_a(T_s - T_a)\left(\frac{86400}{r_h}\right)$$ [kJ m^{-2} day^{-1}]

(see G.S. Campell, An introduction to environmental biophysics, 1977)

IV. Latent Heat Exchange

(IV.1) $$Q_e = f(u)(\rho_{vs} - \rho_{va})$$ [kJ m^{-2} day^{-1}]

ρ_{vs} = vapor density of the surface [kg m^{-3}]
for wet surfaces use equation (IV.3) substituting T_s for T

ρ_{va} = vapor density of the air (see below) [kg m^{-3}]

$f(u)$ = dependence of vapor exchange to windspeed, u [kJ m kg^{-1} day^{-1}]
= $4.58 \times 10^5 (1+u)$

u = average daily windspeed [m s^{-1}]

The empirical term, $f(u)$, is sometimes replaced by a more mechanistic expression for vapor conductance or resistance, r_v [s/m]:

(IV.2) $$Q_e = 2.1 \text{ x } 10^8 \ \frac{(\rho_{vs} - \rho_{va})}{r_v}$$ [kJ m^{-2} day^{-1}]

where the constant 2.1x10^8 has units of [kJ s kg^{-1} day^{-1}]
(see G.S. Campell, An introduction to environmental biophysics, 1977)

The saturation vapor density at some temperature, T, can be estimated with:

(IV.3) $$\rho_v^o = \exp\left(\frac{16.78T - 116.9}{T + 273.3}\right)\left(\frac{1}{(273.15 + T)R}\right)$$ [kg m^{-3}]

T = temperature [^{o}C]
R = the gas constant, 0.4615 [kJ kg^{-1} ^{o}K^{-1}]

The vapor density of the air can be determined using Eq. (IV.2):
(A) Substitute the dew point temperature, T_d, for T.
(B) In a real bind you might assume the minimum daily air temperature, $T_n \approx T_d$.
(C) Use the average daily air temperature to determine ρ_v^o and multiply ρ_v^o by the relative humidity, *RH*.

NOTICE: In many situations sensible and latent heat exchanges are difficult to determine directly because you have to know how hot and wet the surface is and this information is often unavailable. Also, these terms are very sensitive to wind, which is hard to meaningfully characterize. We will investigate how Penman and Priestly-Taylor made approximations to deal with these problems for potential evapotranspiration (*PET*).