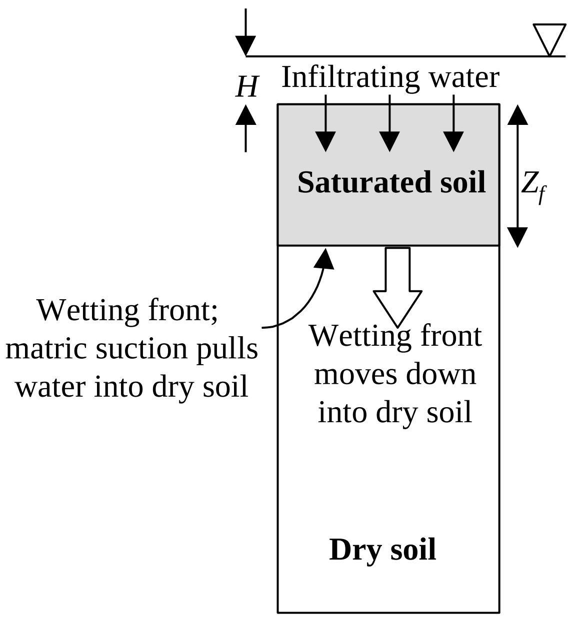
Physical Hydrology for Ecosystems

BEE 3710

Green and Ampt Infiltration: Green, W.H. and G. Ampt. 1911. Studies of soil physics, part I – the flow of air and water through soils. *J. Ag. Sci*. 4:1-24.

The Horton equation captures the basic behavior of infiltration but the physical interpretation of the exponential constant is uncertain. Green and Ampt (1911) presented an approach that is based on fundamental physics and also gives results that match empirical observations. They use the following simplification of infiltration reality:



In reality, there is often not a sharp wetting front and/or the soil above the wetting front may not saturate. The equation to use if you need to consider the most realistic situation is the Richard's equation; Richard's equation is beyond the scope of this class but you should be aware of it.

The problem with all mechanistic infiltration equations is uncertainty about how to generalize to the field or landscape scale, especially with respect to the suction forces at the wetting front. None-the-less, many researchers are embracing these approaches and making good progress so you should have some rudimentary knowledge of, at least, the Green and Ampt concept.

Below is a summary of the relevant Green and Ampt infiltration equations. We will revisit these later in the semester after we are familiar with Darcy's Law. The following equations come from Mein R.G. and C.L. Larson (1973) [Modeling infiltration during a steady rain. *Water Resour. Res.* 9(2): 384-394.] who reduced the Green and Ampt concept to something applicable.

In its simplest form the Green and Ampt equation for infiltration rate, *f*, can be written as:

(1a) $$f = -K_s \frac{dh}{dz} \qquad \frac{dh}{dz} = \text{hydraulic gradient} \qquad [\text{cm}^3\ \text{s}^{-1}\ \text{cm}^{-2}]$$

(1b) $$f = -K_s \frac{h_f - h_o}{Z_f} \qquad [\text{cm}^3\ \text{s}^{-1}\ \text{cm}^{-2}]$$

(1c) assume no ponding, $h_o = 0$ $$f = K_s \frac{|\psi_f| + Z_f}{Z_f} \qquad [\text{cm}^3\ \text{s}^{-1}\ \text{cm}^{-2}]$$

The subscript "*f*" refers to the wetting front and "*o*" refers to the soil surface, e.g., h_f is the hydraulic head at the wetting front (sum of matric forces at the wetting front and the weight of the water above), and h_o is the hydraulic head at the surface (zero, unless there is water ponded on the surface). ψ_f = matric pressure at the wetting front [cm of water], K_S = saturated hydraulic conductivity [cm/hr]. The depth of the wetting front can be related to the cumulative amount of infiltrated water, *F* [cm], by:

(2) $$F = Z_f(\theta_s - \theta_i) \qquad [\text{cm}^3\ \text{s}^{-1}\ \text{cm}^{-2}]$$

where θ_s = saturated moisture content and θ_i = initial moisture content before infiltration began. Rearranging Eq. 2 to solve for Z_f and substituting it into Eq. 1c, the infiltration rate, $f(t)$, becomes:

(3a) $$f(t) = K_s + K_s \frac{|\psi_f|(\theta_s - \theta_i)}{F} \qquad \text{for } t > t_p \qquad [\text{cm}^3\ \text{s}^{-1}\ \text{cm}^{-2}]$$

(3b) $$f(t) = P \qquad \text{for } t \le t_p \qquad [\text{cm}^3\ \text{s}^{-1}\ \text{cm}^{-2}]$$

where: P = rainfall rate [cm hr^{-1}] and t_p is the time when water begins to pond on the surface [hr]. Unfortunately, Eq. 3a does not have time as a variable but instead uses *F*, the cumulative amount of water that has infiltrated. Recognizing that $f = dF/dt$, we can solve Eq. 3 to get the following, somewhat complicated, expression for $F(t)$:

(4) $$t = t_p + \frac{1}{K_s}\left[F - F_p + |\psi_f|(\theta_s - \theta_i)\ln\left(\frac{|\psi_f|(\theta_s - \theta_i) + F_p}{|\psi_f|(\theta_s - \theta_i) + F}\right)\right]$$

where F_p = the amount of water that infiltrates before water begins to pond at the surface [cm] and t_p = the time it takes to have water begin to pond at the surface [hr]. The following are expressions of these quantities:

(5) [from 3a and 3b] $$F_p = \frac{|\psi_f| K_s(\theta_s - \theta_i)}{P - K_s}$$ $t = t_p$ and $P > K_s$

(6) [integrate 3b] $$t_p = \frac{F_p}{P}$$

To determine the amount of infiltration from a rain storm of duration, t_r, and intensity P you will have to first determine the time at which surface ponding occurs (Eqs. 4 & 5). If $t_r < t_p$ or $P < K_s$ then the amount of infiltration, $F = Pt_r$ and the infiltration rate, $f = P$. If $t_d > t_p$, then you will have to use Eq. 4 and find, by trial and error, the value F that gives $t = t_r$. I usually set up an Excel spreadsheet with a column of F, incremented by small amounts, with adjacent columns for t (using Eq. 4) and f (using Eq. 3). Then I can make graphs of infiltration rate or amount verse time.

Example: What's the total runoff and infiltration [cm] from a 2-hour rainfall event with a 0.5 cm/hr intensity? When does runoff begin? The soil's K_s 0.044 cm/hr, $\theta_i = 0.25$ and $\theta_s = 0.50$, and $\psi_f = 22.4$ cm (we could calculate K_s and ψ_f if we know the soil type). What's the infiltration rate at the end of the storm? When you plot f vs t, does the curve look like anything we've seen before?

STEP 1: Calculate t_p and F_p

$$F_p = \frac{|\psi_f| K_s(\theta_s - \theta_i)}{P - K_s} = \frac{(22.4)(0.044)(0.50 - 0.25)}{0.5 - 0.044} = 0.54cm$$

$$t_p = \frac{F_p}{P} = \frac{0.54}{0.5} = 1.08hr$$

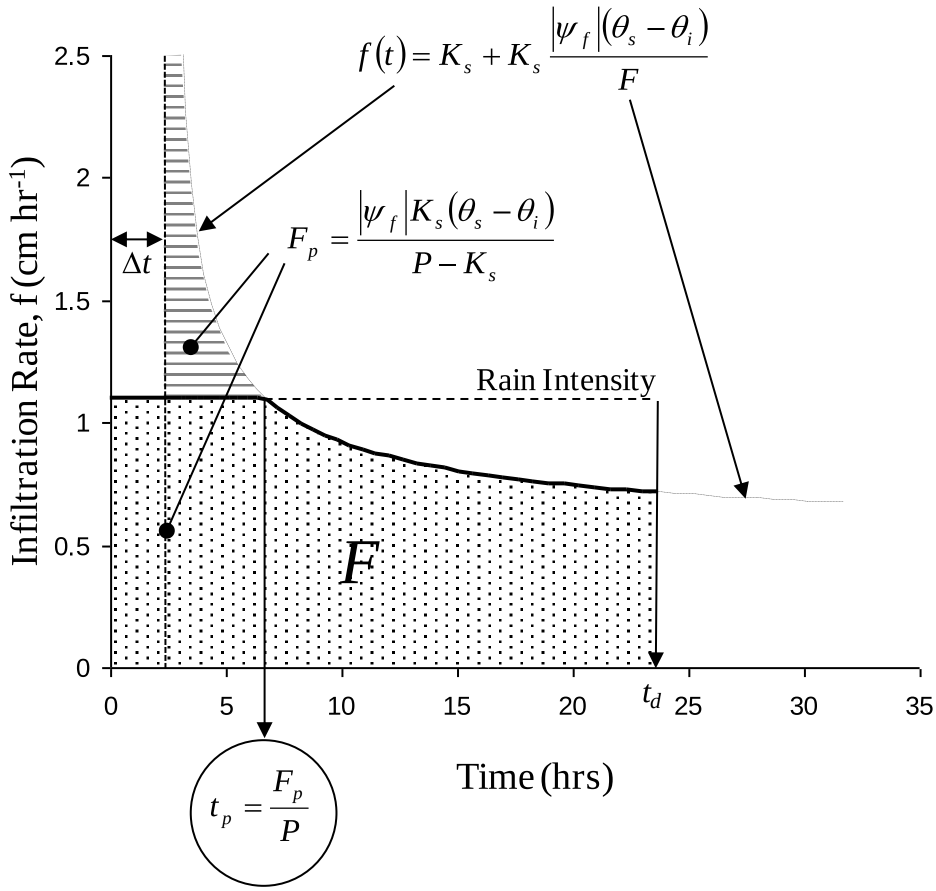
STEP 2: Calculate t and f using Eqs. 4 and 3, respectively

F (cm) (Just Pick These Values)	t (hrs) (Eqs. 4 and 5)	f (cm/hr) (Eq. 3)
0	0	($t < t_p$, Eq. 3b) $P = 0.50$
0.3	($F < F_p$, $t = F/P$) 0.6	($t < t_p$, Eq. 3b) $P = 0.50$
0.54 (= F_p)	1.08 (= t_p)	0.50
0.6	($F > F_p$, Eq. 4) 1.21 *	($t > t_p$, Eq. 3a) 0.45 **
0.7	1.44	0.39
0.8	1.71	0.35
0.9	2.01 (rain is over)	0.31

$$^{*}\ t = 1.08 + \frac{1}{0.044}\left[0.6 - 0.54 + 22.4(0.50 - 0.25)\ln\left(\frac{22.4(0.50 - 0.25) + 0.54}{22.4(0.50 - 0.25) + 0.6}\right)\right] = 1.21hr$$

$$^{**}\ f(t) = K_s + K_s\frac{|\psi_f|(\theta_s - \theta_i)}{F} = 0.044 + 0.044\frac{22.4(0.50 - 0.25)}{0.6}$$

Interpreting Green and Ampt Results:



The figure above identifies the parts of an infiltration-rain intensity analysis with respect to Green and Ampt. The curved dashed line is what the Green and Ampt equation, Eq. 4, mathematically describes and you can see that for times below the time of ponding, t_p, it deviates substantially from the solid line, which is how we hypothesize that the actual infiltration processes occur. Therefore, it is important to remember that the mathematical results for F vs. t, Eq. 4, and f vs t, Eq. 3, for times less than the ponding time, t_p, are incorrect and that, during this perioid, f = rain intensity (P) and $F = Pt$. Always graph your solutions and make sure that your math matches what you conceptually think is happening. For times larger than t_p, the F vs t and f vs t are correct.

One difficulty with Horton's equation is that it is difficult to predict how it would change if the soil were initially a little wet. Built into the Green and Ampt analysis is the math to shift the infiltration capacity curve (the curved dashed line) to account for the initial water content; this is the Δt in the figure:

(6) $$\Delta t = \frac{F_p}{K_s} - \frac{|\psi_f|(\theta_s - \theta_i)}{K_s} \ln\left(1 + \frac{F_p}{\psi_{mf}(\theta_s - \theta_i)}\right) - t_p$$ [cm^3 s^{-1} cm^{-2}]

You don't usually have to calculate Δt because it has been accounted for already in the Green and Ampt solution above (Eq. 4).

Often, K_s and t_p (and by association, F_p) are the only components of the Green and Ampt analysis that need to be calculated, especially in humid, well vegetated areas, like the northeastern U.S., where the soils' saturated hydraulic conductivity is much greater than typical rainfall intensities. Most of the time we find that either our storm intensity is less than K_s or that the storm is over before t_p and, thus, all the rainwater infiltrates.

Practice:

If you would like to investigate a realistically complex problem, consider a 3-hour, 6-cm storm on a clay loam soil (b = 5.2, ψ_e = 26.5 cm, K_s = 0.23 cm hr^{-1}, $\theta_s \approx 0.35$) with an initial moisture content, $\theta_i \approx 0.25$. Use the following expression for the matric potential at the wetting front:

(7)
$$\left|\psi_f\right| = \frac{2b+3}{b+3}\psi_e\left[1-\left(\frac{\theta_i}{\theta_s}\right)^{b+3}\right] \approx \frac{2b+3}{b+3}\psi_e$$

You should find that after 3 hours, ~2.8 cm has infiltrated ($F \approx 2.8$ cm); notice that this means more than half the rain became overland flow.