Daily Potential Evapotranspiration

Notation:  

\( PET, PE, E_o, ET_o \)  
\( T = \) temperature  
\( \rho_v = \) vapor density  
\( \rho^0_v = \) saturation vapor density  
\( e = \) vapor pressure \( = 4.26 \times 10^{-6} \rho_v T \)  
\( \{ T \text{ in } ^{\circ}\text{K} \} \)  

I. Mass Balance

\[ PET = \left( \frac{\rho^0_v - \rho_{va}}{r_v} \right) \frac{1}{\rho_w} \]  
\( \rho^0_v = \) saturated vapor density @ surface  
\( \rho_{va} = \) vapor density of air  
\( \rho_w = \) density of water  
\( r_v = \) resistance to vapor transfer, very sensitive to windspeed  
\( u = \) average windspeed  
\( k = \) von Karman Constant  
\( z_o = \) surface roughness (grass: \( z_o = 0.5 \) ish) \( \sim 0.13h \)  
\( d = \) zero plane displacement \( \sim 0.77h \)  
\( h = \) vegetation height

II. Energy Equations (Note energy is expressed in kJ)

Basic concept:

\[ PET = \frac{Q_e}{\lambda_v \rho_w} \]  
\( Q_e = \) evaporative (latent) heat flux  
\( \lambda_v = \) latent heat of vaporization

Energy balance gives an estimate of \( Q_e \):
\[ Q_e = Q_m - Q_h \]  
\[ Q_m = \text{net radiation energy flux} \quad \text{[kJ m}^{-2} \text{ d}^{-1}] \]
\[ Q_h = \text{Sensible heat flux} \quad \text{[kJ m}^{-2} \text{ d}^{-1}] \]
\[ Q_{\text{rn}} = \text{net radiation energy flux} \quad \text{[kJ m}^{-2} \text{ d}^{-1}] \]
\[ Q_{\text{rn}} = \text{net solar radiation + atmospheric long wave radiation} - \text{terrestrial long wave} \]
\[ T_s = \text{temperature of the surface} \quad \text{[°C]} \]
\[ T_a = \text{air temperature} \quad \text{[°C]} \]
\[ r_h = \text{resistance to heat transfer} \approx r_v \quad \text{[d/m]} \]
\[ C = \text{heat capacity of air} \quad \text{[1.2 kJ m}^{-3} \text{ °C}^{-1}] \]

**Penman** (1948, *Proc. Royal Soc. A194:220*) **Equation** simplifies the energy balance by removing all “surface” terms:

\[
Q_e = \frac{C \left( \rho^o_{va} - \rho_{va} \right)}{\gamma + \Delta} + \Delta Q_m \quad \text{[kJ m}^{-2} \text{ d}^{-1}] 
\]

\[
Q_e = \frac{f(u)(\rho^o_{va} - \rho_{va}) + \Delta Q_m}{\gamma + \Delta} \quad \text{[kJ m}^{-2} \text{ d}^{-1}] 
\]

\[ \rho^o_{va} = \text{saturation vapor density at air temperature} \quad \text{[kg m}^{-3}] \]
\[ \gamma = \text{psychrometric constant} \quad \text{[4.95x10}^{-4} \text{ kg m}^{-3} \text{ °C}^{-1}] \]
\[ \gamma = \frac{C}{\lambda_v} \]
\[ \Delta = \text{slope of the saturation curve on the psychrometric chart} \quad \text{[kg m}^{-3} \text{ °C}^{-1}] \]
\[ \Delta \approx 3.221x10^{-4} \exp(0.8876T^{0.08}) \quad \text{for } 0<T<25^\circ \text{C} \]
\[ \Delta \approx 3.405x10^{-4} \exp(0.0642T) \quad \text{for } T>0^\circ \text{C} \]
\[ f(u) = \text{an empirical wind function that replaces} \frac{\lambda_v}{r_v} \quad \text{[kJ m kg}^{-1} \text{ d}^{-1}] \]
\[ f(u) \approx 4.8x10^3(1+u); u \text{ is the daily average windspeed [m/s] and measured at 2 m.} \]

Note: Dingman's (2002, *Physical Hydrology, 2nd ed*) version of this equation (7-33) is for PET obtained by dividing equation (7a) by \( \lambda_v \rho_w \) (i.e., as in equation (3)) and using conductivity instead of resistance and vapor pressure instead of vapor density. The other apparent differences are mostly notational.

**Penman Approximation:**
\[ \rho_{vs} - \rho_{va} = \rho^o_{vs} - \rho_{va} + \Delta(T_s - T_a) \]
Priestly-Taylor (1972, *Mon. Weather Rev.* 100:81-92) equation simplifies the Penman equation by assuming the at the vapor deficit term and net radiation term are proportional.

\[
\alpha' = \frac{C r_v (\rho_v^a - \rho_v)}{\Delta Q_{rn}} \quad [\text{kJ m}^{-2} \text{d}^{-1}]
\]

The resulting Priestly-Taylor equation uses a constant \( \alpha = 1 + \alpha' \).

\[
Q_e = \alpha \frac{\Delta Q_{rn}}{\gamma + \Delta} \quad [\text{kJ m}^{-2} \text{d}^{-1}]
\]

The common assumption is \( \alpha = 1.26 \).