Physical Hydrology for Ecosystems

BEE 3710

Daily Potential Evapotranspiration

Notation: PET, PE, E_o, ET_o [m d^{-1}]

T = temperature [^oK or ^oC]

ρ_v = vapor density [kg/m^3]

ρ^o_v = saturation vapor density [kg/m^3]

e = vapor pressure = $4.26\times10^{-6}\rho_v T$ [mb] {T in ^oK}

I. Mass Balance

(1)
$$PET = \left(\frac{\rho^o_{vs} - \rho_{va}}{r_v}\right)\frac{1}{\rho_w} \qquad \text{[m/s]}$$

ρ^o_{vs} = saturated vapor density @ surface [kg m^{-3}]

ρ_{va} = vapor density of air [kg m^{-3}]

ρ_w = density of water [10^3 kg m^{-3}]

r_v = resistance to vapor transfer, **very sensitive to windspeed** [d/m]

(2)
$$r_v = \frac{\left(\ln\left(\frac{z-d+z_o}{z_o}\right)\right)^2}{uk^2} \sim \frac{\left(\ln\left(\frac{2}{z_o}\right)\right)^2}{uk^2} \quad / \text{ 86400 s/d}$$

u = average windspeed [m/s]

k = von Karman Constant [0.41]

z_o = surface roughness (grass: z_o = 0.5 ish) ~ $0.13h$ [m]

d = zero plane displacement ~ $0.77h$ [m]

h = vegetation height [m]

II. Energy Equations (Note energy is expressed in kJ)

Basic concept:

(3)
$$PET = \frac{Q_e}{\lambda_v \rho_w} \qquad \text{[m d}^{-1}\text{]}$$

Q_e = evaporative (latent) heat flux [kJ m^{-2} d^{-1}]

(4)
$$= \lambda_v\left(\frac{\rho_{vs} - \rho_{va}}{r_v}\right)$$

λ_v = latent heat of vaporization [2500 kJ kg^{-1}]

Energy balance gives an estimate of Q_e:

(5) $$Q_e = Q_{rn} - Q_h$$ [kJ m^{-2} d^{-1}]

Q_{rn} = net radiation energy flux [kJ m^{-2} d^{-1}]
= net solar radiation + atmospheric long wave radiation – terrestrial long wave
Q_h = Sensible heat flux [kJ m^{-2} d^{-1}]

(6) $$= C\left(\frac{T_s - T_a}{r_h}\right)$$

T_s = temperature of the surface [^{o}C]
T_a = air temperature [^{o}C]
r_h = resistance to heat transfer ≈ r_v [d/m]
C = heat capacity of air [1.2 kJ m^{-3} ^{o}C^{-1}]

Penman (1948, *Proc. Royal Soc.* A194:220) **Equation** simplifies the energy balance by removing all "surface" terms:

(7a) $$Q_e = \frac{C\frac{\left(\rho^o_{va} - \rho_{va}\right)}{r_v} + \Delta Q_{rn}}{\gamma + \Delta}$$ [kJ m^{-2} d^{-1}]

(7b) $$Q_e = \frac{\gamma f(u)\left(\rho^o_{va} - \rho_{va}\right) + \Delta Q_{rn}}{\gamma + \Delta}$$ [kJ m^{-2} d^{-1}]

$\rho^o{}_{va}$ = saturation vapor density at air temperature [kg m^{-3}]
γ ~ psychrometric constant [4.95x10^{-4} kg m^{-3} ^{o}C^{-1}]
= $\frac{C}{\lambda_v}$

Δ = slope of the saturation curve on the psychrometric chart [kg m^{-3} ^{o}C^{-1}]
≈ $3.221x10^{-4}\exp\left(0.8876T^{0.08}\right)$ for 0<T<25^{o}C [kg m^{-3} ^{o}C^{-1}]
≈ $3.405x10^{-4}\exp\left(0.0642T\right)$ for T>0^{o}C [kg m^{-3} ^{o}C^{-1}]

$f(u)$ = an empirical wind function that replaces $\frac{\lambda_v}{r_v}$ [kJ m kg^{-1} d^{-1}]
~ $4.8x10^5\left(1+u\right)$; u is the daily average windspeed [m/s] and measured at 2 m.

Note: Dingman's (2002, *Physical Hydrology*, *2nd ed*) version of this equation (7-33) is for PET obtained by dividing equation (7a) by $\lambda_v \rho_w$ (i.e., as in equation (3)) and using conductivity instead of resistance and vapor pressure instead of vapor density. The other apparent differences are mostly notational.

Penman Approximation: $\rho_{vs} - \rho_{va} = \rho^o_{va} - \rho_{va} + \Delta\left(T_s - T_a\right)$

Priestly-Taylor (1972, *Mon. Weather Rev.* 100:81-92) **equation** simplifies the Penman equation by assuming the at the vapor deficit term and net radiation term are proportional.

(8) $$\alpha' = \frac{\frac{C}{r_v}\left(\rho_{va}^{o} - \rho_{va}\right)}{\Delta Q_{rn}}$$ [kJ m^{-2} d^{-1}]

The resulting Priestly-Taylor equation uses a constant $\alpha = 1 + \alpha'$.

(9) $$Q_e = \alpha \frac{\Delta Q_{rn}}{\gamma + \Delta}$$ [kJ m^{-2} d^{-1}]

The common assumption is $\alpha = 1.26$.