Physical **H**ydrology for **E**cosystems

BEE 3710

<u>Thornthwaite-Mather for *Monthly* Watershed Yield</u>

The following schematic explains how the different conceptual portions of a watershed are combined in the Thornthwaite-Mather model.

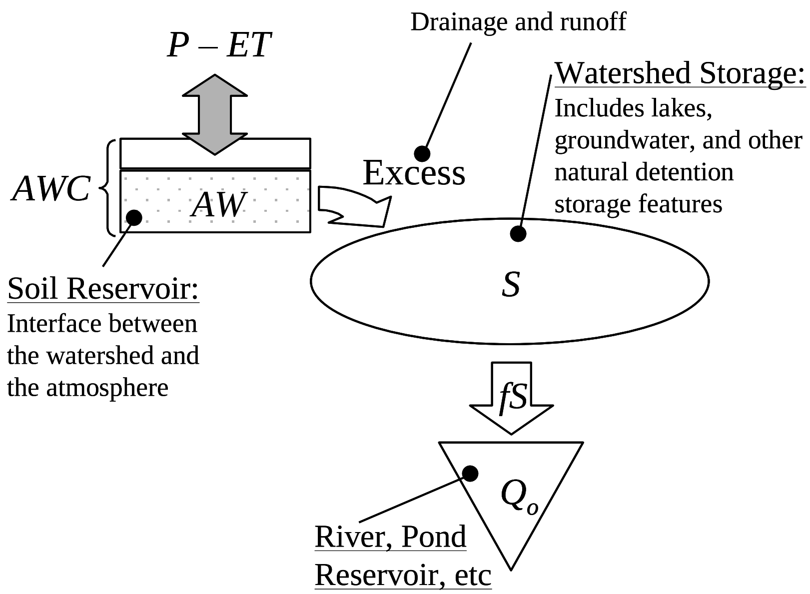

Figure 1. The soil reservoir and how it connects to stream discharge at the watershed outlet, Q_o.

Determining the soil water budget is the most difficult part of the Thornthwaite-Mather method.

Notation:	AWC = Available Water Capacity (soil depth) $\left(\theta_{fc} - \theta_{wp}\right)$	[depth]
	AW = Available Soil Water; (soil depth) $\left(\theta - \theta_{wp}\right)$	[depth]
	ΔP = Net Precipitation; P - PET	[depth]
	P = Precipitation	[depth]
	PET = Potential Evapotranspiration	[depth]

Calculations to determine AW are performed for each month using monthly precipitation (P) and potential evapotranspiration (PET). Excess water, i.e., net precipitation (ΔP) in excess of the soil's water holding capacity (AWC) leaves the soil and is stored in the watershed and eventually released to the river. Table 1 summarizes the calculations.

Table 1. Summary of Thornthwaite-Mather soil-water budget calculations

Situation in the Watershed	*AW*	Excess
• Soil is drying $\Delta P < 0$	$= AW_{t-1} \exp\left(\frac{\Delta P}{AWC}\right)$	$= 0$
• Soil is wetting $\Delta P > 0$ but $AW_{t-1} + \Delta P \le AWC$	$= AW_{t-1} + \Delta P$	$= 0$
• Soil is wetting above capacity $\Delta P > 0$ but $AW_{t-1} + \Delta P > AWC$	$= AWC$	$= AW_{t-1} + \Delta P - AWC$

Watershed Storage and River Discharge:
All *Excess* water, i.e., water above the *AWC*, goes into watershed storage (*S*), which in-turn, feeds river discharge (Q_o) from the watershed.

$$S_t = S_{t-1} + Excess$$

Hydrologists commonly assume that discharge is a constant fraction of watershed storage, especially for groundwater discharge into rivers – this assumption is called the *linear reservoir* assumption.

$$Q_o = fS_t$$

Where f is the reservoir coefficient and $f < 1$ (general case $f \approx 0.5$). If stream flow data are available, f can be empirically determined from hydrograph recession analyses.

Notes on adopting Thornthwaite-Mather for modeling daily hydrological modeling

The Thornthwaite-Mather relationships shown in Table 1 can be readily adapted to modeling daily hydrology simply by dividing the “excess” water into a portion that recharges the groundwater, a.k.a. drainage water, and a portion that is routed as overland flow (See Fig. 2).

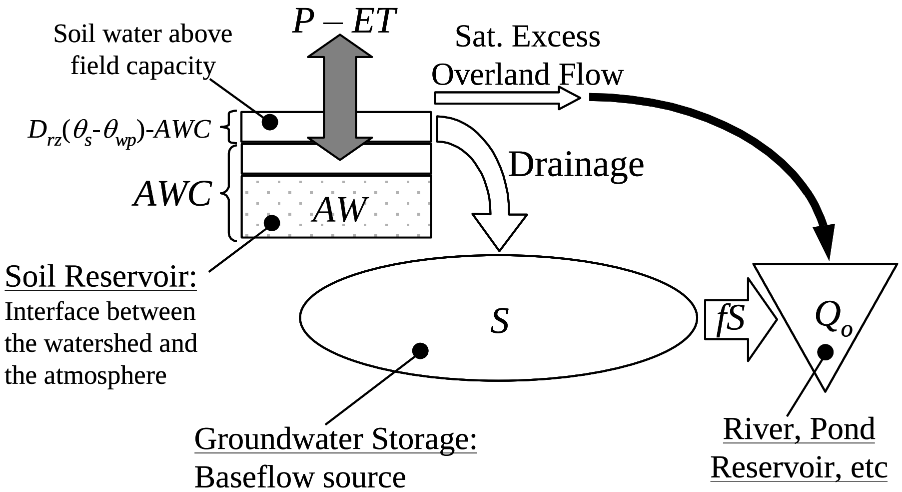

Figure 2. Schematic of a daily soil-water and watershed budget. D_{rz} is root zone depth.

The drainage portion of the excess water is all the water above field capacity but below saturation and the overland flow is all water that exceeds soil saturation (i.e., saturation excess overland flow). Adapting the Thornthwaite-Mather soil-water budget to a daily time step require keeping track of one extra water reservoir, i.e., the soil water above field capacity and below saturation, which we usually assume drains our of the root zone in a day. The simplest assumption is that it drains to the groundwater the same day. Notice if Fig. 2 that the baseflow from the groundwater reservoir is modeled the same way (i.e., as a linear reservoir) that we modeled flow from the more generic storage in the monthly water balance.

The daily Thornthwaite-Mather soil water budget is the basis of many widely used watershed models including SMR (e.g., Frankenberger et al. 1999. *Hydrol. Proc*. 13(6):804) and some recent versions of the widely used “Curve-Number” runoff model (e.g., Lyon et al. 2004. *Hydrol Proc*. 18(15): 2757-2771).

Some Thornthwaite-Mather References:

Bugmann H, Cramer W. 1998. Improving the behavior of forest gap models along drought gradients. *For. Ecol. Mgt*. 103(2-3): 247-263.

Feddema JJ. 1998. Estimated impacts of soil degradation on the African water balance and climate. *Climate Res* 10(2): 127-14.

Leathers DJ, Grundstein AJ, Ellis AW. 2000. Growing season moisture deficits across the northeastern United States. *Climate Res*. 14(1): 43-55.

Liebe, J.R., N. van de Giesen, M. Andreini, M.T. Walter, T.S. Steenhuis. 2009. Determining watershed response in data poor environments with remotely sensed small reservoirs as runoff gauges. *Water Resour. Res.* 45: 2008WR007369

Sridhar, V.. and K.G. Hubbard. 2010. Estimation of the water balance using observed soil water in the Nebraska Sandhills. ASCE J. Hydrologic Engineering 15(1): 70-78.

Steenhuis, T.S. and W.H. Van der Molen. 1986. The Thornthwaite-Mather procedure as a simple engineering method to predict recharge. *J. Hydrol*. 84:221-229.

Thornthwaite, C.W. and J.R. Mather, 1955. *The water balance*. Laboratory of Climatology, No. 8, Centerton NJ.

Varni MR, Usunoff EJ. 1999. Simulation of regional-scale groundwater flow in the Azul River basin, Buenos Aires Province, Argentina. *Hydrogeo. J*. 7(2): 180-187.

Derivation of the Thornthwaite-Mather water budget

Thornthwaite, C.W. and J.R. Mather, 1955. *The water balance*. Laboratory of Climatology, No. 8, Centerton NJ.

Definition of Variables

AW = available water or soil water: $AW = D_{rz}\left(\theta - \theta_{wp}\right)$

AWC = available water capacity: $AWC = D_{rz}\left(\theta_{fc} - \theta_{wp}\right)$

D_{rz} = Depth or the root zone (or depth to restrictive layer)

$PET = \Delta P$ = net potential evapotranspiration, i.e., Precipitation - PET

ET = net actual evapotranspiration

By definition, if there is no precipitation, the decrease in available soil water is equal to the ET:

$$\frac{dAW}{dt} = -ET \tag{1}$$

If we assume a linear relationship between ET and AW and assume that the maximum $ET = PET$ and the maximum $AW = AWC$(see fig.1), then we can estimate ET as:

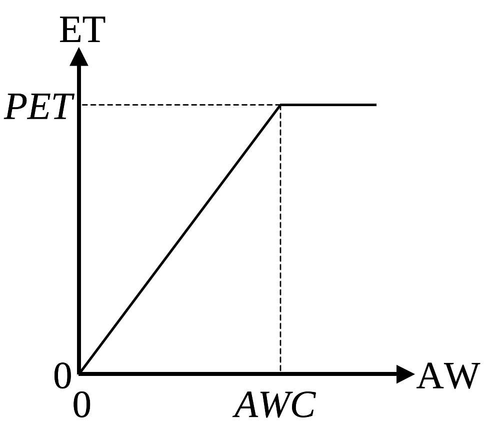

Figure 1. Hypothetical relationship between ET and AW.

$$ET = PET\,\frac{AW}{AWC} \tag{2}$$

In the event of rain, we will assume that the PET demand will first be utilized to evaporate the rainwater before drawing water from the soil, thus the PET is the net PET or PET – precipitation (note that this is a negative number during soil drying). This assumption is justified because presumably rainwater at the soil surface is most readily in contact with the atmosphere and subjected to less matric suction than the soil water. Substituting Eq. (2) into Eq. (1) and separating variables gives:

$$\frac{dAW}{AW} = -\frac{1}{AWC}PETdt \tag{3}$$

Integrating both sides of Eq. (3):

$$\int_{AW_{t-1}}^{AW_t} \frac{dAW}{AW} = -\frac{1}{AWC} \int_{t-1}^{t} PET dt \tag{4}$$

Solution:

$$\ln\left(\frac{AW_t}{AW_{t-1}}\right) = -\frac{\int_{t-1}^{t} PET dt}{AWC} \tag{5}$$

Solve Eq. (5) for AW_t:

$$AW_t = AW_{t-1} \exp\left(-\frac{\int_{t-1}^{t} PET dt}{AWC}\right) \tag{6}$$

Using a discrete, daily time step, Eq. (6) is rewritten as:

$$AW_t = AW_{t-1} \exp\left(-\frac{PET_t}{AWC}\right) \tag{7}$$

Although the Thornthwaite-Mather method is derived here as a soil-drying process in the absence of rain, rainfall can be included by replacing PET_t with a net value, ΔP = Precip – PET; this simply assumes all rain evaporates if PET is large enough. The wetting situation, i.e., $\Delta P > 0$, is trivial to solve (see Table 1).

In some publications Eq. (7) is written as:

$$AW_t = AWC \exp\left(\frac{APWL_t}{AWC}\right) \tag{8}$$

where the accumulated potential water loss, $APWL_t = \int_0^t PET dt = -\sum_{t=0}^{t} PET_t$ during drying periods and is calculated by rearranging Eq. (8) to solve for $APWL_t$ during wetting periods. This expression has recently been abandoned in favor of the simpler Eq. 7.