CULVERTS AND PIPE SPILLWAYS

The hydraulics of pipes can be complicated, but for design purposes we only need to consider a couple basic relationships from fluid mechanics and use a few culvert design rules to avoid situations with unpredictable flow behaviors. In most cases there are three basic flow conditions that engineers consider, pipe flow with a submerged outlet, pipe flow with a free outlet, and orifice flow at the inlet (A, B, and C, below, respectively). Submerged outlet conditions are determined by the depth of flow below the pipe or culvert. To determine whether the culvert will experience pipe or orifice flow, compare the neutral slope, s_n, to the actual slope of the pipe, s; if $s > s_n$ then orifice flow will persist. Conceptually, in a steeply sloped culvert, inertial forces will dominate over frictional forces and the flow will be controlled at the culvert opening (orifice flow). The neutral slope is the slope of energy grade line when the pipe just flows full, i.e., when the momentum due to the inertial force and the momentum loss due to friction are equal, and can generally be approximated with:

$$s_n = K_c \frac{v^2}{2g} \tag{1}$$

where K_c is the friction loss coefficient (tabulated) and v is the flow velocity in the full pipe. In practice engineers sometimes calculate pipe dimensions for both pipe and orifice conditions and design using the large dimension.

Note that if the upstream flow condition is such that the depth above the pipe's entrance invert, H_u, is less than 1.2 time the pipe diameter, the pipe will be partially full throughout and behave as an open channel (Fig. 1).

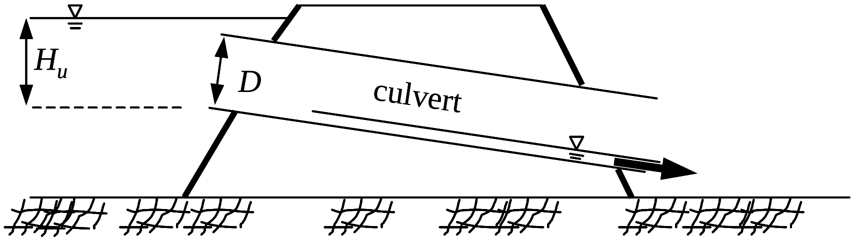


Figure 1: Schematic of upstream flow depth and culver diameter.

Try to ensure that $H_u > 1.5D$ to avoid air entrapment into the culvert barrel.

Relevant descriptions, equations, and tables are provided for each of the three flow conditions.

A. Submerged Outlet (Pipe Flow I)
B. Free Outlet (Pipe Flow II)
C. Orifice Flow Control
D. Culvert Tidbits

A. Submerged Outlet (Pipe Flow I)

The following schematic illustrates the submerged outlet flow condition.

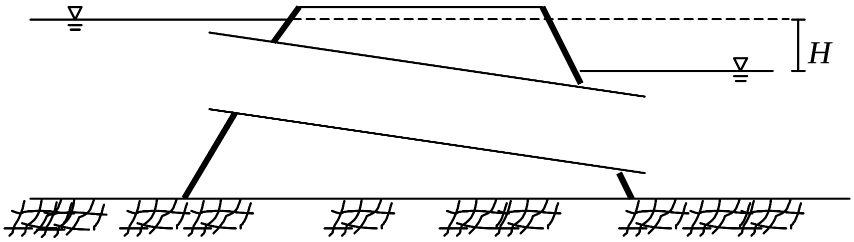


Figure A.1: Schematic of the submerged outlet culvert design condition.

Pipe flow can be calculated with the following mechanistic equation (i.e., as opposed to empirical; units are internally consistent).

$$q = \frac{a\sqrt{2gH}}{\sqrt{1 + K_e + K_c L}} \tag{A.1}$$

where q is the flow rate, a is the pipe cross-sectional area, H is the energy head (defined in Fig. A.1), K_e is the entrance loss (tables follow), K_c is the pipe frictional loss (tables follow), L is the pipe length.

Table C.1 Friction Loss Coefficients for Circular or Square Pipe at Bends

$\frac{R}{D} = \frac{\text{Bend Radius to Pipe Center Line}}{\text{Pipe Diameter}}$	*Bend Coefficient, K_b*	
	45° Bend	*90° Bend*
0.5	0.7	1.0
1	0.4	0.6
2	0.3	0.4
5	0.2	0.3

Source: U.S. Soil Conservation Service (1951). *Engineering Handbook,* Section 5: "Hydraulics." SCS, Washington, DC.

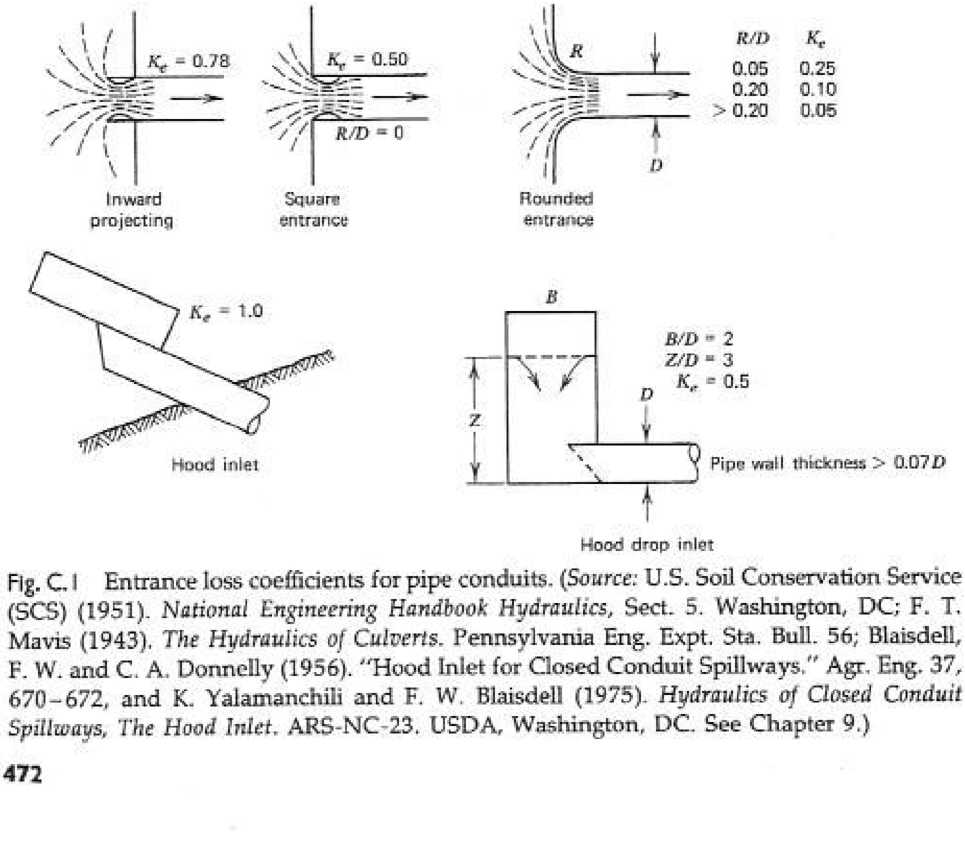


Fig. C.1 Entrance loss coefficients for pipe conduits. (*Source:* U.S. Soil Conservation Service (SCS) (1951). *National Engineering Handbook Hydraulics,* Sect. 5. Washington, DC; F. T. Mavis (1943). *The Hydraulics of Culverts.* Pennsylvania Eng. Expt. Sta. Bull. 56; Blaisdell, F. W. and C. A. Donnelly (1956). "Hood Inlet for Closed Conduit Spillways." Agr. Eng. 37, 670–672, and K. Yalamanchili and F. W. Blaisdell (1975). *Hydraulics of Closed Conduit Spillways, The Hood Inlet.* ARS-NC-23. USDA, Washington, DC. See Chapter 9.)

Table C.2 Head Loss Coefficients for Circular Pipe Flowing Full (SI units)

$$K_c = \frac{1\,244\,522 n^2}{d^{4/3}}, \quad \text{where } d = \text{Diameter (mm)}$$

Pipe Inside Diameter [mm (in.)]	*Flow Area (mm²)*	*Manning Coefficient of Roughness, n*				
		0.010	*0.013*	*0.016*	*0.020*	*0.025*
13 (0.5)	133	4.071	6.881	10.423	16.286	25.447
25 (1)	491	1.702	2.877	4.358	6.810	10.641
51 (2)	2 043	0.658	1.112	1.685	2.632	4.113
76 (3)	4 536	0.387	0.653	0.990	1.546	2.416
102 (4)	8 171	0.261	0.441	0.669	1.045	1.632
127 (5)	12 668	0.195	0.329	0.499	0.780	1.218
152 (6)	18 146	0.153	0.259	0.393	0.614	0.959
203 (8)	32 365	0.104	0.176	0.267	0.417	0.652
254 (10)	50 671	0.0774	0.131	0.198	0.309	0.484
305 (12)	73 062	0.0606	0.102	0.155	0.242	0.379
381 (15)	114 009	0.0451	0.0761	0.115	0.180	0.282
457 (18)	164 030	0.0354	0.0598	0.0905	0.141	0.221
533 (21)	223 123	0.0288	0.0487	0.0737	0.115	0.180
610 (24)	292 247	0.0241	0.0407	0.0616	0.0962	0.150
762 (30)	456 037	0.0179	0.0302	0.0458	0.0715	0.112
914 (36)	656 119	0.0140	0.0237	0.0359	0.0561	0.0877
1219 (48)	1 167 071	0.00956	0.0162	0.0245	0.0382	0.0597
1524 (60)	1 824 147	0.00710	0.0120	0.0182	0.0284	0.0444

Note: K_c (English units) = K_c (SI units)/3.28) ~~d in in~~

Table C.3 Head Loss Coefficients for Square Conduits Flowing Full

$$K_c = \frac{19.60 n^2}{R^{4/3}}, \quad \text{Where } R = \text{Hydraulic Radius (m)}$$

Conduit Size [m × m (ft × ft)]	*Flow Area (m²)*	*Manning Coefficient of Roughness, n*			
		0.012	*0.014*	*0.016*	*0.020*
0.61 × 0.61 (2 × 2)	0.372	0.0347	0.0472	0.0616	0.0963
0.91 × 0.91 (3 × 3)	0.828	0.0203	0.0277	0.0361	0.0564
1.22 × 1.22 (4 × 4)	1.488	0.0138	0.0187	0.0245	0.0382
1.52 × 1.52 (5 × 5)	2.310	0.0103	0.0140	0.0182	0.0285
1.83 × 1.83 (6 × 6)	3.349	0.00800	0.0109	0.0142	0.0222
2.13 × 2.13 (7 × 7)	4.537	0.00653	0.00889	0.0116	0.0181
2.44 × 2.44 (8 × 8)	5.954	0.00545	0.00742	0.00970	0.0152
2.74 × 2.74 (9 × 9)	7.508	0.00467	0.00636	0.00831	0.0130
3.05 × 3.05 (10 × 10)	9.303	0.00405	0.00551	0.00720	0.0113

B. Free Outlet (Pipe Flow II)

The following schematic illustrates the free outlet flow condition with pipe flow in the culvert barrel.

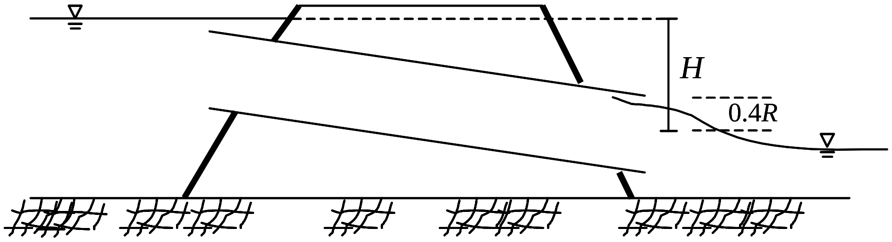


Figure B.1: Schematic of the free outlet culvert design condition

To determine discharge, use Eq. (A.1); note, however, that the definition of *H* has changed (Fig. B.1). *R* is the culvert radius (*D*/2).

C. Orifice Flow Control

Figure C.1 illustrates the orifice flow condition. The flow condition persists when the culvert is especially steep or short and is sometimes referred to as “hydraulically short.”

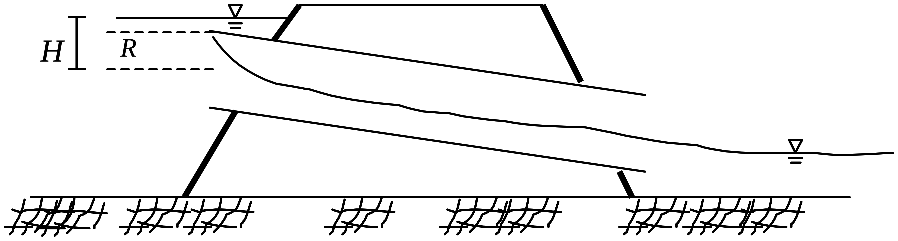


Figure C.1: Schematic of the orifice flow culvert design condition

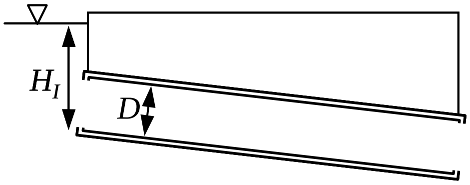
$$q = aC\sqrt{2gH} \tag{A.1}$$

where q is the flow rate, a is the pipe cross-sectional area, H is the energy head (defined in Fig. C.1), C is a head loss coefficient; for sharp inlets this is called the *vena contracta* and can be determined from first principles C = 0.611.

D. Culvert Tidbits

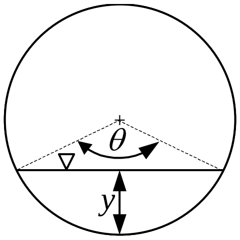
Sizes: Culverts are primarily plastic, concrete, or metal: all three come in 3” intervals from 12” to 36” and 6” intervals up to 96”+. Plastic and metal culverts are available in smaller sizes--Plastic culverts: 2” to 12” diameters in 2” intervals; Metal culverts: 6”, 8” and 10” diameters.

Inlet Conditions: (see figure for variable definitions)
Whether or not the inlet will allow air into the culvert or pipe can be anticipated by the ratio of the depth to the inlet invert (invert = bottom of pipe), H_I, the pipe diameter, D.



$\frac{H_I}{D} > 1.5$	Culvert inlet will be submerged
$\frac{H_I}{D} < 1.5$	Air will probably get into the culvert
$\frac{H_I}{D} < 1.2$	Culvert will most likely run as an open channel

Partially Full Culverts: (see figure for variable definitions)



$$A = \frac{D^2}{8}\left(\theta - \sin(\theta)\right) \qquad \theta = 2\tan^{-1}\left(\frac{\sqrt{\left(D/2\right)^2 - \left(D/2 - y\right)^2}}{D/2 - y}\right) \qquad y < D/2$$

$$R = \frac{D}{4}\left(1 - \frac{\sin(\theta)}{\theta}\right) \qquad \theta = 2\pi + 2\tan^{-1}\left(\frac{\sqrt{\left(D/2\right)^2 - \left(D/2 - y\right)^2}}{D/2 - y}\right) \qquad y > D/2$$

References:

For further information:

Chin, D.A. *Water Resources Engineering*. Prentice Hall. Upper Saddle River. pp. 750.

[*]Chow, V.T. 1959. *Open Channel Hydraulics*. McGraw-Hill Company, New York. pp. 680.

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[*]Montes, S. *Hydraulics of Open Channel Flow*. ASCE Press, Reston. pp. 697.

[†]Schwab, G.O., D.D. Fangmeier, W.J. Elliot, R.K. Frevert. 1993. *Soil and Water Conservation Engineering*, 4th Ed. John Wiley & Sons, Inc. New York. pp.508.

[†]Tollner, E.W. 2002. *Natural Resources Engineering*. Iowa State Press, Ames. pp. 576.

[*] Particularly good books for open channels

[†] These texts were previously used for this course