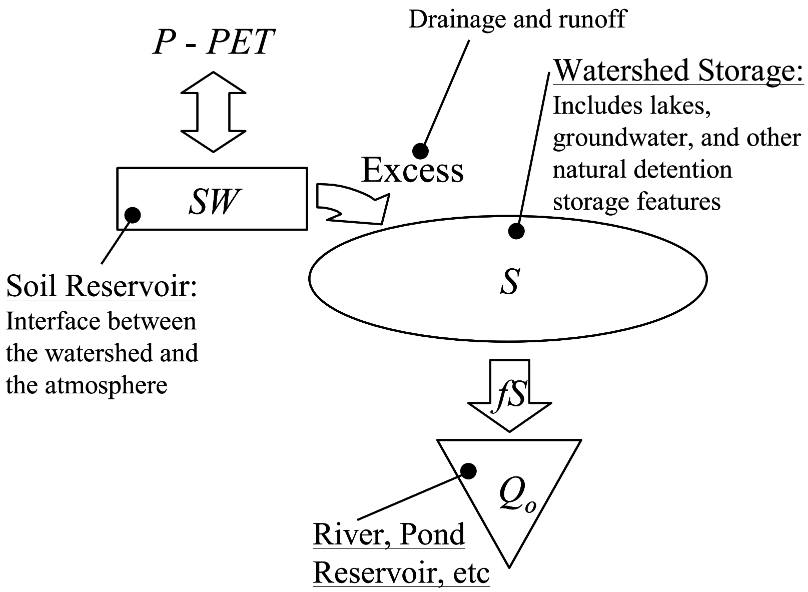
Watershed Engineering

Thornthwaite-Mather for *Monthly* Watershed Yield

The following schematic explains how the different conceptual portions of a watershed are combined in the Thornthwaite-Mather model.



Soil Reservoir: Determining the soil water budget is the most difficult part of the Thornthwaite-Mather method. It requires keeping track of the accumulated potential water loss (*APWL*) and the amount of water in the soil (*SW*).

Notation:	*AWC* = Available Water Capacity (field capacity-wilting point)(soil depth)	[depth]
	SW = Available Soil Water (i.e., above wilting pt.)	[depth]
	ΔP = Net Precipitation; *P* - *PET*	[depth]
	P = Precipitation	[depth]
	PET = Potential Evapotranspiration	[depth]

Calculations to determine *SW* and *APWL* are performed for each month using monthly precipitation (*P*) and potential evapotranspiration (*PET*). Excess water, i.e., net precipitation (ΔP) in excess of the soil's water holding capacity (*AWC*) leaves the soil and is stored in the watershed and eventually released to the river. The table below summarizes the calculations.

Situation in the Watershed	*SW*	Excess
• <u>Soil is drying</u> $\Delta P < 0$	$= SW_{t-1} \exp\left(\frac{\Delta P_t}{AWC}\right)$	$= 0$
• <u>Soil is wetting</u> $\Delta P > 0$ but $SW_{t-1} + \Delta P \leq AWC$	$= SW_{t-1} + \Delta P$	$= 0$
• <u>Soil is wetting above capacity</u> $\Delta P > 0$ but $SW_{t-1} + \Delta P > AWC$	$= AWC$	$= SW_{t-1} + \Delta P - AWC$

Watershed Storage and River Discharge:
All *Excess* water, i.e., water above the *AWC*, goes into watershed storage (*S*), which in-turn, feeds river discharge (Q_o) from the watershed.

$$S_t = S_{t-1} + Excess$$

Hydrologists commonly assume that discharge is a constant fraction of watershed storage, especially for groundwater discharge into rivers – this assumption is called the *linear reservoir* assumption.

$$Q_o = fS_t$$

Where f is the reservoir coefficient and $f < 1$ (general case $f \approx 0.5$). If data are available, f can be empirically determined.

Some Thornthwaite-Mather References:

Bugmann H, Cramer W. 1998. Improving the behaviour of forest gap models along drought gradients. *For. Ecol. Mgt*. 103(2-3): 247-263.

Feddema JJ. 1998. Estimated impacts of soil degradation on the African water balance and climate. *Climate Res* 10(2): 127-14.

Leathers DJ, Grundstein AJ, Ellis AW. 2000. Growing season moisture deficits across the northeastern United States. *Climate Res*. 14(1): 43-55.

Steenhuis, T.S. and W.H. Van der Molen. 1986. The Thornthwaite-Mather procedure as a simple engineering method to predict recharge. *J. Hydrol*. 84:221-229.

Thornthwaite, C.W. and J.R. Mather, 1955. *The water balance*. Laboratory of Climatology, No. 8, Centerton NJ.

Varni MR, Usunoff EJ. 1999. Simulation of regional-scale groundwater flow in the Azul River basin, Buenos Aires Province, Argentina. *Hydrogeo. J*. 7(2): 180-187.

Derivation of the Thornthwaite-Mather water budget
SW = available water or soil water
AWC = available water capacity
PET = net potential evapotranspiration
ET = net actual evapotranspiration

By definition, if there is no precipitation, the decrease in available soil water is equal to the ET

$$\frac{dSW}{dt} = ET = PET\,\frac{SW}{AWC} \tag{1}$$

If we assume a linear relationship between *ET* and *AW* and assume that the maximum *ET* = *PET* and the maximum *SW* = *AWC* then.

$$ET = PET\,\frac{SW}{AWC} \tag{2}$$

In the event of rain, we will assume that the *PET* demand will first be utilized to evaporate the rainwater before drawing water from the soil, thus the *PET* is the net *PET* or *PET* – precipitation. This assumption is justified because presumably rainwater at the soil surface is most readily in contact with the atmosphere and subjected to less matric suction than the soil water. Substituting Eq. (2) into Eq. (1) and separating variables gives:

$$\frac{dSW}{SW} = \frac{1}{AWC}PETdt \tag{3}$$

Integrating both sides:

$$\int_{SW_{t-1}}^{SW_t} \frac{dSW}{SW} = \frac{1}{AWC}\int_{t-1}^{t} PETdt \tag{4}$$

Solution:

$$\ln\left(\frac{SW_t}{AWC}\right) = \frac{\int_{t-1}^{t} PETdt}{AWC} \tag{5}$$

Solve Eq. (5) for SW_t:

$$SW_t = AWC\exp\left(\frac{\int_{t-1}^{t} PETdt}{AWC}\right) \tag{6}$$

Using a discrete, daily time step, Eq. (6) is rewritten as:

$$SW_t = AWC\exp\left(\frac{\Delta P}{AWC}\right) \tag{7}$$

Where ΔP = Net Precipitation; *P* - *PET*