

Ecohydrology

Some Useful Daily Environmental Energy Equations:

I. Solar Radiation

$$(I.1) \quad Q_s = (1 - A)(1 - F_c)T_t S_o \quad [\text{kJ m}^{-2} \text{ day}^{-1}]$$

T_t = atmospheric transmissivity (cloudy \approx 0.6, sunny \approx 0.84). [-]

A = albedo (\sim 0.05 water, \sim 0.2 vegetation, \sim 0.3 soil, \sim 0.9 new snow) [-]

F_c = forest cover [-]

S_o = the potential solar radiation

$$(I.2) \quad S_o = \frac{117500}{\pi} \left[\cos^{-1}(\tan(\delta) \tan(\lambda)) \sin(\lambda) \sin(\delta) + \cos(\lambda) \cos(\delta) \sin(\cos^{-1}(\tan(\delta) \tan(\lambda))) \right] \quad [\text{kJ m}^{-2} \text{ day}^{-1}]$$

λ = geographic latitude. [radians]

δ = solar declination, or the latitude at which the sun is directly overhead

$$(I.3) \quad \delta = 0.4102 \sin \left[\frac{\pi}{180} (J - 80) \right] \quad [\text{radians}]$$

J = the day of the year (Julian date, e.g., Jan 1 = 1, Jan 2 = 2, ... Dec 31 = 365)

Atmospheric Transmissivity (Campbell-Bristow eqn):

$$(I.4) \quad T_t = 0.75 \left[1 - \exp \left(- \frac{B}{S_{o30}} (T_x - T_n)^2 \right) \right] \quad [-]$$

S_{o30} = the potential solar radiation 30 days ago. [$\text{MJ m}^{-2} \text{ day}^{-1}$]

T_x , T_n = Maximum and minimum daily temperatures respectively. [$^{\circ}\text{C}$]

B = a constant loosely correlated with season and latitude, λ .

summer: $B = 0.282\lambda^{-0.431}$ [-]

winter: $B = 0.170\lambda^{-0.979}$ [-]

Summer is defined as the 90 days before and after the summer solstice

II. Long Wave Radiation

Stefan-Boltzmann Equation

$$(II.1) \quad Q_l = \varepsilon \sigma T_s^4 \quad [\text{kJ m}^{-2} \text{ day}^{-1}]$$

$$T_s = \text{temperature} \quad [^{\circ}\text{K}]$$

$$\varepsilon = \text{emissivity} \quad [-]$$

$$\sigma = \text{Stephan-Boltzmann Constant, } 4.90 \times 10^{-6} \quad [\text{kJ m}^{-2} \text{ } ^{\circ}\text{K}^{-4} \text{ day}^{-1}]$$

$$(5.67 \times 10^{-8} \text{ J m}^{-2} \text{ } ^{\circ}\text{K}^{-1} \text{ s}^{-1})$$

Terrestrial Long Wave Radiation (kJ m⁻² day⁻¹):

$$(II.2) \quad Q_t = \varepsilon_s \sigma T_s^4 \quad [\text{kJ m}^{-2} \text{ day}^{-1}]$$

$$T_s = \text{terrestrial surface temperature} \quad [^{\circ}\text{K}]$$

$$\varepsilon_s = \text{surface emissivity} \approx 0.95 - 0.98 \quad [-]$$

Atmospheric Long Wave Radiation (kJ m⁻² day⁻¹):

$$(II.3) \quad Q_a = \varepsilon_{ac} \sigma T_a^4 \quad [\text{kJ m}^{-2} \text{ day}^{-1}]$$

$$T_a = \text{the cloud or air temperature} \quad [^{\circ}\text{K}]$$

$$\varepsilon_a = \text{Clear sky emissivity} \quad \varepsilon_a = 0.72 + 0.005T_a \quad [-]$$

$$T_a = \text{the air temperature (in emissivity equation)} \quad [^{\circ}\text{C}]$$

$$\varepsilon_{ac} = \text{Cloudy sky emissivity} \quad \varepsilon_{ac} = (1 - 0.84C)\varepsilon_a + 0.84C \quad [-]$$

$$C = \text{cloudiness} \quad [\text{fraction}]$$

III. Sensible Heat Exchange

$$(III.1) \quad Q_h \approx C_a (T_s - T_a) \left(\frac{f(u)}{\lambda_v} \right) \quad [\text{kJ m}^{-2} \text{ day}^{-1}]$$

$$T_s = \text{temperature of the surface} \quad [^{\circ}\text{C}]$$

$$T_a = \text{temperature of the air} \quad [^{\circ}\text{C}]$$

$$C_a = \text{The heat capacity of air, } 1.25 [\text{kJ m}^{-3} \text{ } ^{\circ}\text{C}^{-1}]$$

$$\lambda_v = \text{latent heat of vaporization, } 2500 \quad [\text{kJ kg}^{-1}]$$

$$f(u) = \text{dependence of sensible heat exchange to windspeed, } u \quad [\text{kJ m kg}^{-1} \text{ day}^{-1}]$$

$$= 5.3(1 + u)$$

$$u = \text{average daily windspeed} [\text{m s}^{-1}]$$

The empirical term, $f(u)/\lambda$, is sometimes replaced by a more mechanistic expression for thermal conductance or resistance, r_h (see G.S. Campell, An introduction to environmental biophysics, 1977)

IV. Latent Heat Exchange

$$(IV.1) \quad Q_e = f(u)(\rho_{vs} - \rho_{va}) \quad [\text{kJ m}^{-2} \text{ day}^{-1}]$$

ρ_{vs} = vapor density of the surface [kg m⁻³]

for wet surfaces use equation (IV.2) substituting T_s for T

ρ_{va} = vapor density of the air (see below) [kg m⁻³]

$f(u)$ = dependence of vapor exchange to windspeed, u [kJ m kg⁻¹ day⁻¹]

$$= 5.3(1 + u)$$

u = average daily windspeed [m s⁻¹]

The empirical term, $f(u)$, is sometimes replaced by a more mechanistic expression for vapor conductance or resistance, r_v , and the latent heat of vaporization, λ (see G.S. Campell, An introduction to environmental biophysics, 1977)

The saturation vapor density at some temperature, T , can be estimated with:

$$(IV.2) \quad \rho_v^o = \exp\left(\frac{16.78T - 116.9}{T + 273.3}\right) \left(\frac{1}{(273.15 + T)R}\right) \quad [\text{kg m}^{-3}]$$

T = temperature [°C]

R = the gas constant, 0.4615 [kJ kg⁻¹ °K⁻¹]

The vapor density of the air can be determined using Eq. (IV.2):

(A) Substitute the dew point temperature, T_d , for T .

(B) In a real bind you might assume the minimum daily air temperature, $T_n \approx T_d$.

(C) Use the average daily air temperature to determine ρ_v^o and multiply ρ_v^o by the relative humidity, RH .

NOTICE: In many situations sensible and latent heat exchanges are difficult to determine directly because you have to know how hot and wet the surface is and this information is often unavailable. Also, these terms are very sensitive to wind, which is hard to meaningfully characterize. We will investigate how Penman and Priestly-Taylor made approximations to deal with these problems for potential evapotranspiration (*PET*).