Ecohydrology

Daily Potential Evapotranspiration

Notation:  

\[ \text{PET} = PE = E_o = ET_o \]  \[ \text{[m d}^{-1}] \]

\[ T = \text{temperature} \quad \text{[} \circ \text{K or } \circ \text{C}] \]

\[ \rho_v = \text{vapor density} \quad \text{[kg/m}^3] \]

\[ \rho_o^v = \text{saturation vapor density} \quad \text{[kg/m}^3] \]

\[ e = \text{vapor pressure} = 4.26 \times 10^{-6} \rho_v T \quad \text{[mb]} \]

\[ T \text{ in } \circ \text{K} \]

I. Mass Balance

\[ \text{PET} = \left( \frac{\rho_o^v - \rho_{va}}{r_v} \right) \frac{1}{\rho_w^v} \quad \text{[m/s]} \]

\[ \rho_o^v = \text{saturated vapor density @ surface} \quad \text{[kg m}^{-3}] \]

\[ \rho_{va} = \text{vapor density of air} \quad \text{[kg m}^{-3}] \]

\[ \rho_w = \text{density of water} \quad \text{[10}^3 \text{ kg m}^{-3}] \]

\[ r_v = \text{resistance to vapor transfer, very sensitive to windspeed} \quad \text{[d/m]} \]

\[ r_v = \left( \ln \left( \frac{z - d + z_o}{z_o} \right) \right)^2 \sim \left( \ln \left( \frac{2}{z_o} \right) \right)^2 \quad \text{X 86400 s/d} \]

\[ u = \text{average windspeed} \quad \text{[m/s]} \]

\[ k = \text{von Karman Constant} \quad \text{[0.41]} \]

\[ z_o = \text{surface roughness (grass: } z_o = 0.5 \text{ ish) } - 0.13h \quad \text{[m]} \]

\[ d = \text{zero plane displacement } - 0.77h \quad \text{[m]} \]

\[ h = \text{vegetation height} \quad \text{[m]} \]

II. Energy Equations (Note energy is expressed in kJ)

Basic concept:

\[ \text{PET} = \frac{Q_e}{\lambda_v \rho_w} \quad \text{[m d}^{-1}] \]

\[ Q_e = \text{evaporative (latent) heat flux} \quad \text{[kJ m}^{-2} \text{ d}^{-1}] \]

\[ \lambda_v = \text{latent heat of vaporization} \quad \text{[2500 kJ kg}^{-1}] \]

Energy balance gives an estimate of \( Q_e \):
\( Q_e = Q_m - Q_h \) \[\text{[kJ m}^{-2}\text{ d}^{-1}]\]

\( Q_m = \) net radiation energy flux \[\text{[kJ m}^{-2}\text{ d}^{-1}]\]
\( Q_h = \) net solar radiation + atmospheric long wave radiation – terrestrial long wave \[\text{[kJ m}^{-2}\text{ d}^{-1}]\]

\( T_s = \) temperature of the surface \[\text{[°C]}\]
\( T_a = \) air temperature \[\text{[°C]}\]
\( r_h = \) resistance to heat transfer \( \approx r_v \) \[\text{[d/m]}\]
\( C = \) heat capacity of air \[\text{[1.2 kJ m}^{-3}\text{ °C}^{-1}]\]

**Penman** (1948, *Proc. Royal Soc.* A194:220) **Equation** simplifies the energy balance by removing all “surface” terms:

\[
Q_e = \frac{C \left( \rho_{va}^o - \rho_{va} \right) + \Delta Q_m}{\gamma + \Delta}
\]

\[\text{[kJ m}^{-2}\text{ d}^{-1}]\]

\( \rho_{va}^o = \) saturation vapor density at air temperature \[\text{[kg m}^{-3}]\]
\( \gamma = \) psychrometric constant \[\text{[4.95\times10}^{-4}\text{ kg m}^{-3}\text{ °C}^{-1}]\]

\( \gamma \approx \frac{C}{\lambda_v} \)

\( \Delta = \) slope of the saturation curve on the psychrometric chart \[\text{[kg m}^{-3}\text{ °C}^{-1}]\]

\( \approx 3.221\times10^{-4} \exp\left(0.087^{0.8876}\right) \) for \( 0<T<25\text{°C} \)

\( \approx 3.405\times10^{-4} \exp\left(0.0642T\right) \) for \( T<0\text{°C} \)

\( f(u) = \) an empirical wind function that replaces \( \frac{\lambda_v}{r_v} \) \[\text{[kJ m kg}^{-1}\text{ d}^{-1}]\]

\( \sim 5.3(1+u) ; u \) is the daily average windspeed [m/s] and measured at 2 m height.

Note: Dunne and Leopold's (1978, *Water in Environmental Planning*) version of this equation (4-22) is for PET obtained by dividing equation (7) by \( \lambda_r \rho_w \) (equation (4)) and using units of cal, g, days, and cm instead of kJ, kg, s, and m respectively. The Penmann approximation falls out in Dunne and Leopold's approach and is replaced by an empirical relationship, (4-22).

**Penman Approximation:**

\( \rho_{vs} - \rho_{va} = \rho_{va}^o - \rho_{va} + \Delta(T_s - T_a) \)
Priestly-Taylor (1972, Mon. Weather Rev. 100:81-92) equation simplifies the Penman equation by assuming the at the vapor deficit term and net radiation term are proportional.

\[
\alpha' = \frac{C}{r_v} \frac{\rho_v \rho_v^r - \rho_{va}}{\Delta Q_{rn}} \quad [\text{kJ m}^{-2} \text{ d}^{-1}]
\]

The resulting Priestly-Taylor equation uses a constant \( \alpha = 1 + \alpha' \).

\[
Q_e = \alpha \frac{\Delta Q_{rn}}{\gamma + \Delta} \quad [\text{kJ m}^{-2} \text{ d}^{-1}]
\]

The common assumption is \( \alpha = 1.26 \).