Ecohydrology

Daily Evapotranspiration via Penman-Montheith

Notation: ET = Evapotranspiration = $Q_e/(\lambda_v \cdot \rho_w)$ [m d^{-1}]
λ_v = latent heat of vaporization [2500 kJ kg^{-1}]
ρ_w = density of water [10^3 kg m^{-3}]
T = temperature [^{o}K or ^{o}C]
ρ_v = vapor density [kg/m^3]
ρ^o_v = saturation vapor density [kg/m^3]
e = vapor pressure = $4.26\text{x}10^{-6}\rho_v T$ [mb]
{T in ^{o}K}

Penman-Monteith Equation (Monteith, J.L. 1965. Evaporation and environment. In: Proc. 19th Symposium Soc. Exp. Bio. P. 205-233)

(1)
$$Q_e = \frac{\Delta Q_{rn} + C_a \dfrac{\left(\rho^o_{va} - \rho_{va}\right)}{r_a} +}{\gamma + \Delta\left(1 + {}^{r_c}\!/\!_{r_a}\right)} \qquad \text{[kJ m}^{-2}\text{ d}^{-1}\text{]}$$

ρ^o_{vs} = saturated vapor density @ canopy surface [kg m^{-3}]
ρ_{va} = vapor density of air [kg m^{-3}]
γ ~ psychrometric constant [$4.95\text{x}10^{-4}$ kg m^{-3} ^{o}C^{-1}]
= $\dfrac{C}{\lambda_v}$
Δ = slope of the saturation curve on the psychrometric chart [kg m^{-3} ^{o}C^{-1}]
$\approx 3.221x10^{-4}\exp\left(0.8876T^{0.08}\right)$ for 0<T<25^{o}C [kg m^{-3} ^{o}C^{-1}]
$\approx 3.405x10^{-4}\exp\left(0.0642T\right)$ for T<0^{o}C [kg m^{-3} ^{o}C^{-1}]

r_a = atmospheric resistance to vapor transfer, **very sensitive to windspeed** [d/m]

(2)
$$r_a = \frac{\ln\left(\dfrac{z-d+z_h}{z_h}\right)\ln\left(\dfrac{z-d+z_m}{z_m}\right)}{uk^2} \sim \frac{\left(\ln\left(\dfrac{2}{z_m}\right)\right)^2}{uk^2} \quad \text{X 86400 s/d}$$

u = average windspeed [m/s]
k = von Karman Constant [0.41]
z = measurement height [m]
z_m = momentum roughness parameter ≈ 0.13-0.2h [m]
z_h = heat roughness parameter ≈ 0.2z_m [m]
d = zero plane displacement ~ 0.77h [m]
h = vegetation height [m]

NOTE: because the sensitivity of Eq. (2) to wind, the Penman-Monteith equation is often implemented over short time-steps (minutes to hours) and summed to get a total for a day.

r_c = canopy resistance to vapor transfer, **very sensitive to windspeed** [time m^{-1}]

(3) $$r_c = \frac{r_{leaf}}{f_{sh} LAI}$$

f_{sh} = fractions of canopy in shade, sparse veg. = 1, full canapy = 1 [-]

LAI = leaf area index, leaf area per unit area of ground [m^2 m^{-2}]

Leaf Resistance (most of this originates with Jarvis 1976)

Basic concept, minimum leaf resistance (species specific) is scaled by unitless factors (*f*) to account for stomatal resistance incurred due to various environmental characteristics:

(4) $$r_{leaf} = \frac{r_{\min}}{f_S\, f_T\, f_{\Delta\rho}\, f_\theta\, f_{other}}$$ [time m^{-1}]

r_{min} = Minimum leaf resistance (see table below) [time m^{-1}]

f_S = dependence on solar radiation [-]

(5) $$= \frac{12.78 S_{in}}{11.57 S_{in} + 104.4}$$ [from Stewart 1988 via Dingman 2002]

S_{in} = incoming solar radiation [KJ m^{-2} d^{-1}]

f_T = dependence on air temperature (there are relationships for soil temp too) [-]

(6) $$= \begin{cases} 0 & | \, T_a < 0 \\ \dfrac{T_a (40 - T_a)^{1.18}}{691} & | \, 0 \le T_a \le 40 \\ 0 & | \, T_a > 40 \end{cases}$$ [from Stewart 1988 via Dingman 2002]

(7) $$= 0.08 T_a - 0.0016 T_a^2$$ [Dickinson et al. 1991 via Wigmosta et al. 1994]

T_a = air temperature [^{o}C]

$f_{\Delta\rho}$ = dependence on the vapor pressure deficit [-]

(8) $$= \begin{cases} 1 - 66.6 \Delta\rho_{va} & | \, \Delta\rho_{va} \le 0.01152 \\ 0.233 & | \, \Delta\rho_{va} > 0.01152 \end{cases}$$ [from Stewart 1988 via Dingman 2002]

$\Delta\rho_{va}$ = vapor density deficit (sat. vapor desnity – air vapor density: $\rho_{va}{}^{o}$-ρ_{va}) [kg m^{-3}]

f_θ = dependence on soil moisture (there are relationships for soil tension too) [-]

$$(9) \qquad = \begin{cases} 0 & |\,\theta < \theta_{wp} \\ \dfrac{\theta - \theta_{wp}}{\theta_{fc} - \theta_{wp}} & |\,\theta_{wp} \le T_a \le \theta_{fc} \\ 1 & |\,\theta_{fc} > 40 \end{cases} \quad \text{[from Feddes et al. 1978 via Wigmosta et al. 1994]}$$

θ = volumetric soil water content [m^3 m^{-3}]

Alternatively, $\theta - \theta_{wp}$ =available water and $\theta_{fc} - \theta_{wp}$ =available water capacity
NOTE: wilting point (wp) and field capacity (fc) are convenient thresholds to approximate when stomates are fully closed and open, respectively, but may be plant specific.

F_{other} = dependence on other factors, eg. Carbon dioxide [-]

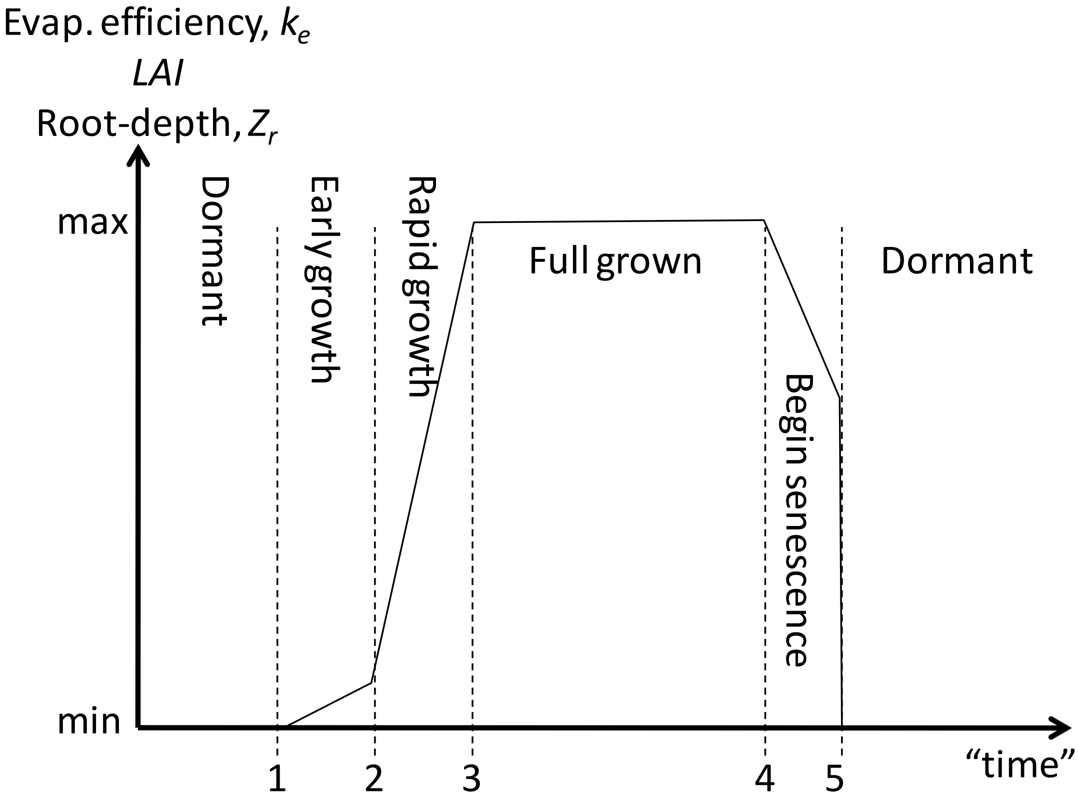
Table 1. Some vegetation/land cover parameters adopted from the Community Climate Model (CCM)*:

	Albedo	**Max *LAI***	**Min *LAI***	$r_{\min}$ [s m^{-1}]
Crop/mixed farming	0.20	6	0.5	120
Short grass	0.20	2	0.5	200
Evergreen (needle)	0.14	6	0.5	200
Deciduous (needle)	0.14	6	1	200
Deciduous (leaf)	0.18	6	1	200
Evergreen (leaf)	0.12	6	0.5	150
Tall grass	0.19	6	0.5	200
Desert	0.30	0	0	200
Tundra	0.20	6	0.5	200
Irrigated crop	0.18	6	0.5	200
Semi-desert	0.26	6	0.5	200
Ice	0.70	0	0	200
Wetland	0.12	6	0.5	200
Fresh water	0.14	0	0	200
Ocean	0.14	0	0	200
Evergreen (shrub)	0.14	6	0.5	200
Deciduous (shrub)	0.18	6	0.5	200
Mixed woodland	0.15	6	0.5	200

*Dickinson, R.E., A. Henderson-Sellers, P.J. Kennedy. 1993. Biosphere-atmosphere transfer scheme (BATS) version 1e as coupled to the NCAR Community Climate Model. NCAR Technical Note NCAR/TN-387+STR

Leaf Area Index:

There are two primary modeling strategies (i.e., not using some kind of measurement) based on fixed time or thermal time, but, both have similar functional shapes (see figure below).



Although here we are interested in the *LAI*, similar modeling approaches could be adopted to account for root depth, i.e., soil depth from which plants uptake water, which affects the AWC, or a general evaporation efficiency, which can be used to scale PET to account for plant development.

The primary issue here is to estimate LAI for Eq. (3). Stewart et al. (1988) used the following time-based estimates for Thetford Forest in the UK:

$$(10) \qquad LAI = \begin{cases} (192.6 - 0.061J)/100 & | \, 0 \le J \le 151 \\ (-12.2 + 1.286J)/100 & | \, 152 \le J \le 224 \\ (290.9 - 0.061J)/100 & | \, 225 \le J \le 233 \\ (504.2 - 0.973J)/100 & | \, 234 \le J \le 316 \\ (215.1 - 0.071J)/100 & | \, 317 \le J \le 365 \end{cases} \qquad \text{[based on Beadle et al. 1982]}$$

Obviously, this approach is going to be somewhat different from location to location.

J = Julian day or day of the year [day]

We developed a thermal time *LAI* model that was developed for the Soil Moisture Routing Model (SMR aka SMDR) (e.g., Frankenberger et al. 1999, Easton et al. 2007) based loosely on data from Goudriann and van Laar (1994). Thermal time for day *t* is:

$$(11) \quad D_t = \begin{cases} T_a - T_b \mid T_a \geq T_b \\ 0 \quad \mid T_a < T_b \end{cases} \quad [-]$$

Thermal time is kept by accumulating degree-days of daily thermal time, *DD*. The numbered threshold points in the plant development in the figure above are determined based on accumulated thermal time.

Growth begins (1): In temperate areas, it is common to assume growth starts when the average five-day temperature is above the base temperature (see table 2).

Table 2 outlines degree-day thresholds in percent of maximum cumulative heat units (a.k.a. potential heat units) for different vegetation/land covers.

Table 2. Base temperature, key threshold thermal-times (% of maximum cumulative heat units) corresponding the figure above, and maximum cumulative heat units for various vegetation/land cover types.

	Base temp. T_b (^{o}C)	**Rapid growth 2 (%)**	**Full growth 3 (%)**	**Begin Senescence 4 (%)**	**Maximum cumulative heat units (deg-days)**
Deciduous forest / mixed forest /shrubland	1	10	22.5	90	2500
Evergreen forest	0	7.5	12.5	95	2500
Natural grasslands	-1	5	10	95	3000
Hay / fallow /pastures	-1	5	10	95	3000
Row crops / small grains	5	15	40	90	2000
Recreational grasses	10	7.5	17.5	85	2500

Dormancy (5): Dormancy can be initiated in a variety of ways, including, crop harvest, frost, or a maximum cumulative heat units. One approximation for (killing) frost conditions is when the mean five-day temperature is lower than -3^{o}C.

Leaf area index changes between $LAI_{\min}$ and $LAI_{\max}$ (Table 1) as a function of thermal-time *DD* (degree-days). The growth rate, α_g, is calculated as:

(12) $$\alpha_g = \begin{cases} \frac{1}{DD_4} DD & |\, Early-growth \\ \frac{DD_2}{DD_4} + \frac{1}{DD_4}\frac{(DD_4 - DD_2)}{(DD_3 - DD_2)}(DD - DD_2) & |\, Rapid-growth \\ 1 & |\, Full-grown \\ 1 - 0.6\frac{1}{DD_{\max} - DD_4}(DD - DD_4) & |\, Senescence \\ 0 & |\, Dormant \end{cases}$$

The *LAI* is:

(13) $$LAI = LAI_{\min} + \alpha_g (LAI_{\max} - LAI_{\min})$$ [-]

This growth model can also be used for root growth between $Z_{\text{r-min}}$ and $Z_{\text{r-max}}$, recognizing that some ecosystems will develop over a number of years to a maximum level and should probably not be returned to a minimum depth at the end of each season. Similarly, for very long simulations, the landscape may go through long-term ecological succession changes that need extra consideration.

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