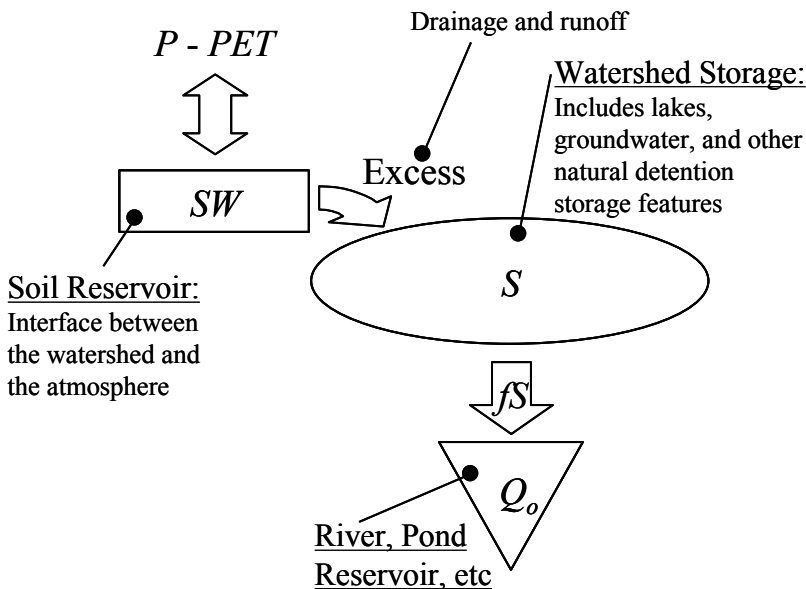


Ecohydrology

Thornthwaite-Mather for *Monthly* Watershed Yield

The following schematic explains how the different conceptual portions of a watershed are combined in the Thornthwaite-Mather model.



Soil Reservoir: Determining the soil water budget is the most difficult part of the Thornthwaite-Mather method. One simplifying assumption is that rainfall will evaporate before soil water (SW). Therefore, we can consider the difference in precipitations (P) and potential evapotranspiration (PET), i.e., the net P (or, conversely, the net PET) – we will use the notation ΔP .

Notation:	AWC = Available Water Capacity (field capacity-wilting point)(soil depth)	[depth]
	SW = Available Soil Water (i.e., above wilting pt.)	[depth]
	ΔP = Net Precipitation; $P - PET$	[depth]
	P = Precipitation	[depth]
	PET = Potential Evapotranspiration	[depth]

Calculations to determine SW are performed for each month using monthly precipitation (P) and potential evapotranspiration (PET). Excess water, i.e., net precipitation (ΔP) in excess of the soil's water holding capacity (AWC) leaves the soil and is stored in the watershed and eventually released to the river. The table below summarizes the calculations.

Situation in the Watershed	SW	Excess
<ul style="list-style-type: none"> • <u>Soil is drying</u> $\Delta P < 0$ 	$= SW_{t-1} \exp\left(\frac{\Delta P_t}{AWC}\right)$	$= 0$
<ul style="list-style-type: none"> • <u>Soil is wetting</u> $\Delta P > 0$ but $SW_{t-1} + \Delta P \leq AWC$ 	$= SW_{t-1} + \Delta P$	$= 0$
<ul style="list-style-type: none"> • <u>Soil is wetting above capacity</u> $\Delta P > 0$ but $SW_{t-1} + \Delta P > AWC$ 	$= AWC$	$= SW_{t-1} + \Delta P - AWC$

Watershed Storage and River Discharge:

All *Excess* water, i.e., water above the *AWC*, goes into watershed storage (*S*), which in-turn, feeds river discharge (Q_o) from the watershed.

$$S_t = S_{t-1} + Excess$$

Hydrologists commonly assume that discharge is a constant fraction of watershed storage, especially for groundwater discharge into rivers – this assumption is called the *linear reservoir* assumption.

$$Q_o = fS_t$$

Where *f* is the reservoir coefficient and $f < 1$ (general case $f \approx 0.5$). If data are available, *f* can be empirically determined.

Some Thornthwaite-Mather References:

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Derivation of the Thornthwaite-Mather water budget SW = available water or soil water AWC = available water capacity PET = net potential evapotranspiration ET = net actual evapotranspiration

By definition, if there is no precipitation, the decrease in available soil water is equal to the ET

$$\frac{dSW}{dt} = ET = PET \frac{SW}{AWC} \quad (1)$$

If we assume a linear relationship between ET and AW and assume that the maximum $ET = PET$ and the maximum $SW = AWC$ then.

$$ET = PET \frac{SW}{AWC} \quad (2)$$

In the event of rain, we will assume that the PET demand will first be utilized to evaporate the rainwater before drawing water from the soil, thus the PET is the net PET or $PET - \text{precipitation}$. This assumption is justified because presumably rainwater at the soil surface is most readily in contact with the atmosphere and subjected to less matric suction than the soil water. Substituting Eq. (2) into Eq. (1) and separating variables gives:

$$\frac{dSW}{SW} = \frac{1}{AWC} PET dt \quad (3)$$

Integrating both sides:

$$\int_{SW_{t-1}}^{SW_t} \frac{dSW}{SW} = \frac{1}{AWC} \int_{t-1}^t PET dt \quad (4)$$

Solution:

$$\ln\left(\frac{SW_t}{AWC}\right) = \frac{\int_{t-1}^t PET dt}{AWC} \quad (5)$$

Solve Eq. (5) for SW_t :

$$SW_t = AWC \exp\left(\frac{\int_{t-1}^t PET dt}{AWC}\right) \quad (6)$$

Using a discrete, daily time step, Eq. (6) is rewritten as:

$$SW_t = AWC \exp\left(\frac{\Delta P}{AWC}\right) \quad (7)$$

Where ΔP = Net Precipitation; $P - PET$