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Comparison of TOPMODEL and STOPMODEL: Conceptualizations, Derivations, and Implementations

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Abstract

TOPMODEL (Beven and Kirkby 1979) is a popular physically-based watershed model capable of simulating variable source area processes (Anderson et al. 1997) but its conceptualization is uncertain with respect to shallow, interflow driven systems (Scanlon et al. 2000, Frankenberger et al. 1999, Moore and Thompson 1996). STOPMODEL follows much of the TOPMODEL approach but explicitly addresses shallow systems by replacing "depth to watertable," z, with moisture content, θ (Walter et al. 2002b). The resulting expressions are very similar to the common TOPMODEL expression but maintain conceptual appropriateness for shallow systems. The similarity in the final expressions combined with the highly calibrated nature of most TOPMODEL applications may explain why TOPMODEL appears to work well across a wide range of hydrological systems, i.e. its general equations can be derived from different assumptions about the modeled system. This is a parallel derivation of both models. The most substantial differences are in the applications of the two models. In the applications it becomes obvious that STOPMODEL explicitly routes shallow subsurface flow and TOPMODEL does not.

Derivations of Basic Models Principles

TOPMODEL	STOPMODEL	
Both models start with a basic expression of quasi-steady state water balance at a point, *i*, in the landscape and assume the flux through a unit contour length is proportional to a transmissivity and, by the kinematic approximation, the local land slope:		
$a_i R = q_i$	$a_i R = q_i$	(1)
$q_i = T_i \tan \beta$	$q_i = T_i \tan \beta$	(2)
Where R is the recharge due to drainage from the root zone. Equation (2) is Darcy's law and the transmissivity is defined with the following:	Where R is the soil recharge and can be generally taken as the excess precipitation (precipitation – evapotranspiration) for humid, well vegetated areas (Walter et al. 2002a). Equation (2) is assumed to be valid for both saturated and unsaturated conditions and the transmissivity is defined with the following:	
$T_i = \int_{z_i}^{Z} K_s(z)dz$	$T_i = K(\theta)D_i$	(3)
Where z is the depth to the watertable and Z is the depth to the bottom of the groundwater. The saturated hydraulic conductivity is most commonly taken as an exponentially decreasing function with depth, although other functions are occasionally used:	Where D_i is the depth of the soil. The hydraulic conductivity is taken as an exponential with respect to soil moisture deficit, $\Omega(\theta)$ (e.g. Hillel 1980).	
$K_s(z) = K_o \exp\left(-\frac{z}{m}\right)$	$K(\theta) = K_s \exp(-\kappa\,\Omega)$ $\Omega = 1 - \frac{\theta - \theta_d}{\theta_s - \theta_d}$	(4)
Where m is a calibrated scaling parameter and K_o is the saturated conductivity at the surface. Local	Where θ is the soil moisture and θ_s and θ_d are saturated and aid-dry soil moisture contents	

transmissivity is found by integrating equation (4) as shown in equation (3) and assuming that Z is very big so exp(-Z/m)≈0; this is one aspect of this derivation of TOPMODEL that is conceptually inappropriate for shallow soils:

$$T_i = T_o \exp\left(-\frac{z_i}{m}\right)$$
$$T_o = mK_o$$

Where T_o is typically calibrated independently of m. Combining equation (1), (2), and (5) and solving for the local depth to groundwater:

respectively. The parameter κ is a soil parameter; similar to that used in SMR (Frankenberger et al. 1999) and by Bresler et al. (1978) and Steenhuis and Van der Molen (1986). Local transmissivity is:

$$T_i = T_s \exp(-\kappa\,\Omega) \qquad (5)$$
$$T_s = D_i K_s$$

Where both D_i and K_s are measurable parameters; K_s is the bulk saturated conductivity for the entire soil profile. Combining equation (1), (2), and (5) and solving for the local moisture defecit:

$$z_i = -m\ln\left(\frac{a_i R}{T_o \tan\beta}\right) \qquad \Omega_i = -\frac{1}{\kappa}\ln\left(\frac{a_i R}{T_s \tan\beta}\right) \qquad (6)$$

Equation (6) can be rearranged to solve for ln(R).

$$\ln(R) = \ln\left(\frac{a_i}{T_o \tan\beta}\right) - \frac{z_i}{m} \qquad \ln(R) = \ln\left(\frac{a_i}{T_s \tan\beta}\right) - \kappa\Omega_i \qquad (7)$$

Integrating equation (6) gives basin averages of the state variables, z_i and Ω_i, for TOPMODEL and STOPMODEL respectively. Then, using equation (7) and recognizing that recharge, R, is spatially averaged (i.e. not distributed), the following expressions are derived relating the local state variable to the basin average:

$$\bar{z} = \frac{1}{A}\int z_i dA \qquad \overline{\Omega} = \frac{1}{A}\int \Omega_i dA \qquad (8)$$
$$\bar{z} = -\frac{m}{A}\int \ln(\varsigma R) dA \qquad \overline{\Omega} = -\frac{1}{A\kappa}\int \ln(\varsigma R) dA$$
$$\bar{z} = -\frac{m}{A}\int [\ln(\varsigma) + \ln(R)].dA \qquad \overline{\Omega} = -\frac{1}{A\kappa}\int [\ln(\varsigma) + \ln(R)].dA$$
$$\bar{z} = -m[\ln(R) + \gamma] \qquad \overline{\Omega} = -\frac{1}{\kappa}[\ln(R) + \gamma]$$
$$\frac{1}{m}(\bar{z} - z_i) = \ln(\varsigma) - \gamma \qquad \kappa(\overline{\Omega} - \Omega_i) = \ln(\varsigma) - \gamma$$

Where:

$$\varsigma = \frac{a_i}{T_o \tan\beta} \qquad \varsigma = \frac{a_i}{T_s \tan\beta} \qquad (9)$$
$$\gamma = -\frac{1}{A}\int \ln(\varsigma) dA \qquad \gamma = -\frac{1}{A}\int \ln(\varsigma) dA$$

For simplification, a relative storage deficit, δ, can be defined to further reduce equations (8) (Ambroise et al. 1996a):

$$\delta = \frac{z}{m} \qquad \delta = \kappa\Omega \qquad (10)$$
$$\bar{\delta} - \delta_i = \ln(\varsigma) - \gamma \qquad \bar{\delta} - \delta_i = \ln(\varsigma) - \gamma$$

The base flow, Q_b, can be expressed in terms of the above expressions by summing up drainage along all the channel reaches, each of length l_j.

$Q_b = \sum l_j q_j$	$Q_b = \sum l_j q_j$	(11)
$Q_b = \sum l_j T_{oj} \tan\beta \exp(-\delta_j)$	$Q_b = \sum l_j T_{oj} \tan\beta \exp(-\delta_j)$	
$Q_b = \sum l_j a_j \exp(-\bar{\delta})\exp(-\gamma_l)$	$Q_b = \sum l_j a_j \exp(-\bar{\delta})\exp(-\gamma_l)$	
$Q_b = Q_o \exp(-\bar{\delta})$	$Q_b = Q_o \exp(-\bar{\delta})$	
$Q_o = A\exp(-\gamma_l)$	$Q_o = A\exp(-\gamma_l)$	

Where Q_o is the flow when the average basin-wide storage deficit is zero.

Both models end up with identical expressions of base flow and therefore may be fit to observed recession flow data equally well. Note that TOPMODEL has two calibration parameters, *m* and T_o, whereas STOPMODEL has one, κ. The physical interpretation of *m* is unclear and T_o is rarely corroborated with direct field measurements; both are calibrated using observed recession flow behavior and/or direct optimization of TOPMODEL results. There are several, independently determined estimates for the soil parameter, κ, which is not unique to STOPMODEL (Bresler et al. 1978, Steenhuis and Van der Molen 1986, Frankenberger et al. 1999). Based on Steenhuis et al. (1999), a low value of κ may represent hillslope flow controlled by macropores. Note that T_s, STOPMODEL's corollary to T_o, is easily conceptualized and can be readily estimated from tabulated Soil Survey information. While a physical interpretation of T_o is also conceptually simple, independent parameterization is not simple; this is one reason why it is typically calibrated.

Model Implementations

Because TOPMODEL is a loose conglomerate of ideas, it is difficult to make concrete comparisons, i.e. each application is somewhat unique. With that said, figure 1: shows the general conceptual designs for TOPMODEL and STOPMODEL and illuminates some of the main differences and similarities. When applying TOPMODEL, the depth to watertable, *z*, is often replaced by saturated storage deficit, *S*, in equations (8) (Beven and Wood 1983, Hornberger et al. 1985, Ambroise et al. 1996b). A similar substitution can be made for STOPMODEL for the soil moisture deficit, Ω. In both models a basin-average mass balance is used determine the average storage deficit, $\bar{S}$, which is directly related to the storage at any point in the basin, S_i, by equations (8). Values of $S_i \leq 0$ indicate a saturated contributing runoff area. The basin water balance for each model is described below.

TOPMODEL	STOPMODEL	
$\bar{S}_t = \bar{S}_{t-1} + Q_b - R$	$\bar{S}_t = \bar{S}_{t-1} + Q_b + L + ET - R$	(12)
Where *t* is the time step, Q_b is the base flow to the stream, and *R* is recharge from the soil reservoir (Figure 1a). The recharge is usually calculated by using a linear reservoir model for drainage from the soil reservoir.	Where *t* is the time step, Q_b is the base flow to the stream, *L* is the leakage through the shallow restrictive layer to deep groundwater, *ET* is evapotranspiration, and *R* is recharge (Figure 1b). Recharge is the excess precipitation:	
$R = \alpha V_{drain}$	$R = P - PET$ for *P-PET*>0 $R = 0$ for *P-PET*≤0	(13)

Where α is a reservoir constant and V_{drain} is the water stored in the soil reservoir in excess of "field capacity." The reservoir constant is often considered inversely proportional to $\bar{S}$ and a characteristic vadose-zone residence time (Hornberger et al. 1985, Ambroise et al. 1996b). Sdrain is handled as a unique reservoir that is filled from the general soil reservoir, V_{soil}, whenever V_{soil}, exceeds a "field capacity" threshold. The general soil reservoir is subjected to infiltrating rain and snowmelt, P, and evapotranspiration, ET:

$$V_{soil} = P - ET$$

Any storage in excess of saturation is routed to the stream as saturation excess runoff and any remaining water above "field capacity" is routed to V_{drain}. Sometimes a third reservoir is employed to account for interception and surface storage. ET is calculated by assuming PET for V_{soil} above some threshold and decreasing linearly with V_{soil} below this threshold to zero at a "wilting point." This is the same assumption used in the Thronthwaite-Mather (1955) approach (Steenhuis and Van der Molen 1986).

Where P is precipitation and snowmelt and PET is potential evapotranspiration. Note that if $R \geq 0$, then $ET = 0$ in equation (12) because the evaporative demand will have been met by new P. The leakage through the restrictive layer, L, occurs only when $\bar{S}$ is below a threshold value that indicates the soil is drainable; this threshold would be associated with something akin to "field capacity." L is modeled using Darcy's law assuming gravity forces dominate and drainage assumed to recharge the deep ground water reserve, V_{gw}:

$$L = K_r \tag{14}$$

Evapotranspiration, ET, can be modeled using an approach similar to the Thornthwaite-Mather (1955) approach as described by Steenhuis and Van der Molen (1986). Note that ET will only be explicitly accounted for in equation (12), when recharge, $R = 0$ (equation (13)).

For both models the baseflow, Q_b, is calculated using equations (11).

After performing the basin mass balance as described above, equations (8) are used to check for localized areas of surface saturation, saturation excess water is routed to the stream such that the localized storage deficits, S_i, are zero. The average storage deficit is recalculted after routing saturation excess and model proceeds to the next time step's mass balance.

For TOPMODEL, streamflow is the sum of Q_b and saturation excess runoff. In STOPMODEL, Q_b is shallow interflow and the streamflow is the sum of Q_b, saturation excess runoff, and groundwater flow, Q_{gw}. The groundwater flow can be estimated via a variety of approaches, perhaps the most simple are the TOPMODEL approach as adapted by Scanlon et al. (2000) or a linear reservoir approach similar to SMR (Frankenberger et al. 1999). The latter is interesting because it provides an interesting corollary to TOPMODEL. The shaded regions in figure 1 show where each model places its linear reservoir.

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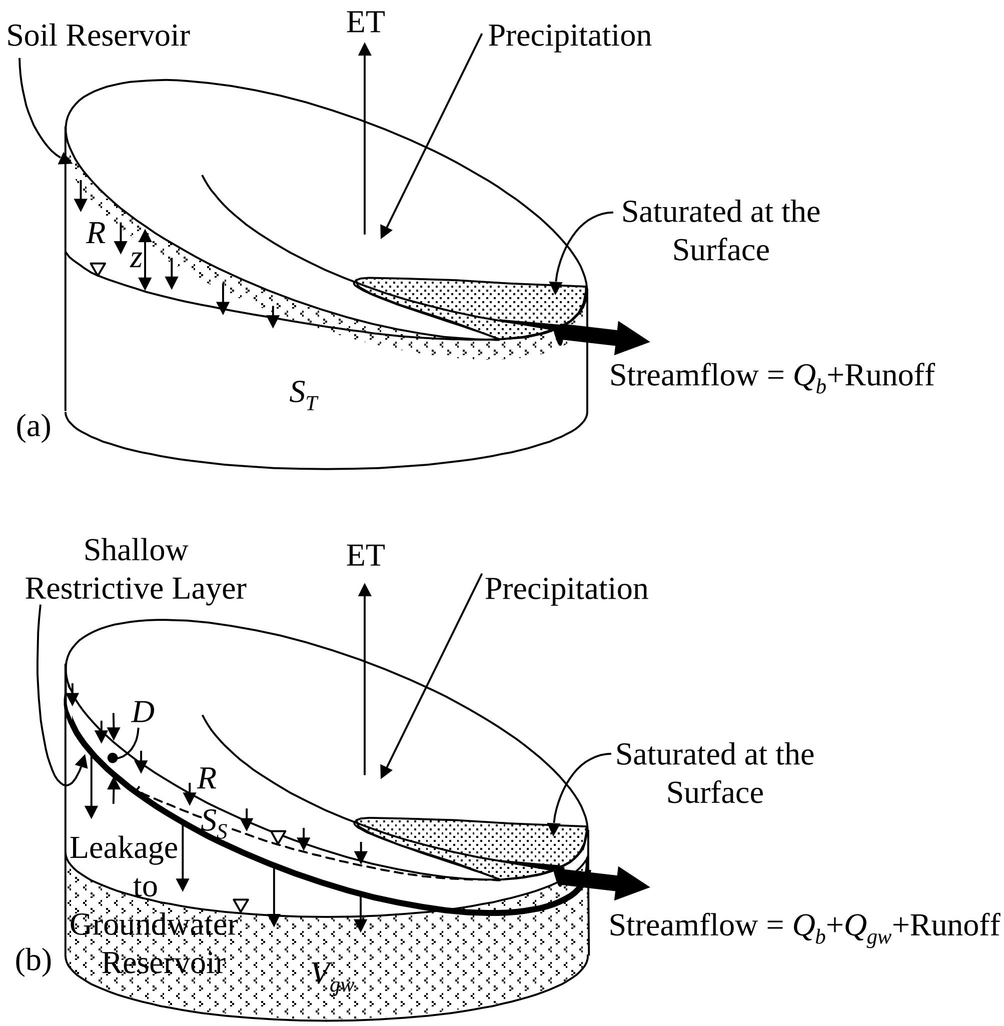

Figure 1: Schematics of the conceptual design for TOPMODEL (a) and STOPMODEL (b). S_T and S_S indicate the physical locations where TOPMODEL and STOPMODEL, respectively, store the water that controls variable source areas. Also note the different interpretations of recharge, *R*.